# MATHEMATICS

## Grade 11

## Part - II

Educational Publications Department

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# The National Anthem of Sri Lanka

Sri Lanka Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Sundara siri barinee, surendi athi sobamana Lanka
Dhanya dhanaya neka mal palaturu piri jaya bhoomiya ramya
Apa hata sepa siri setha sadana jeewanaye matha
Piliganu mena apa bhakthi pooja Namo Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Oba we apa vidya
Obamaya apa sathya
Oba we apa shakthi
Apa hada thula bhakthi
Oba apa aloke
Apage anuprane
Oba apa jeevana we
Apa mukthiya oba we
Nava jeevana demine, nithina apa pubudukaran matha
Gnana veerya vadawamina regena yanu mana jaya bhoomi kara
Eka mavakage daru kela bevina
Yamu yamu vee nopama
Prema vada sema bheda durerada
Namo, Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha

**අපි වෙමු එක මවකගෙ දරුවෝ**
**එක නිවසෙහි වෙසෙනා**
**එක පාටැති එක රුධිරය වේ**
**අප කය තුළ දුවනා**

**එබැවිනි අපි වෙමු සොයුරු සොයුරියෝ**
**එක ලෙස එහි වැඩෙනා**
**ජීවත් වන අප මෙම නිවසේ**
**සොඳින සිටිය යුතු වේ**

**සැමට ම මෙත් කරුණා ගුණෙනී**
**වෙළී සමගි දමිනී**
**රන් මිණි මුතු නො ව එය ම ය සැපතා**
**කිසි කල නොම දිරනා**

**ආනන්ද සමරකෝන්**

*ம்ஒரு தாய் மக்கள் நாமாவோ*
*ம்மீஒன்றே நாம் வாழும் இல்*

*ம்ன்றே உடலில் ஓடுந*
*ம்ஒன்றே நம் குருதி நிற*

*ம்அதனால் சகோதரர் நாமாவோ*
*ம்ஒன்றாய் வாழும் வளரும் நா*
*மீனே்றாய் இவ் இல்லினிந*
*வாழ்தல் வேண்டுமன்றோ மமேீந*

*ாவரும் அன்பு கருணையுடன்ய*
*ஒற்றுமை சிறக்க வாழ்ந்திடுதல்*
*பொன்னும் மணியும் முத்துமல்ல - அதுவே*
*.ான்று மழியாச் செல்வமன்றோய*

*ஆனந்த சமரக்கோன்*
*.புப்கவிதையின் பெயர்*

## Message of the Hon. Minister of Education

Sri Lanka is fortunate to be among the few countries in the world that offer free education. Our expectation is to ensure the privilege of free education by all means. One of the pioneer missions under free education in Sri Lanka is providing free textbooks with the aim of making the subject matter under a prescribed syllabus, available for all the children in schools in a formal way. You are a fortunate child to receive the benefits of free education.

Sri Lanka should step forward on par with the other countries in this "century of knowledge". Our sole aim is to present the society a student generation empowered with updated knowledge appropriate for the motherland and the world at large. The government has spent an enormous expenditure for providing free textbooks for a student population over four million. Your duty and responsibility is to use this book and obtain the prescribed knowledge in order to reach the top rung in the ladder of education. Further, you have to read supplementary books to gather knowledge as well. You have to be prepared to learn to learn. This book is given to you with the solemn expectation that you would acquire a strong foundation to become a citizen useful for the country and for this era.

**AkilaViraj Kariyawasam**
Minister of Education

# Foreword

The process of facilitating learning or the acquisition of knowledge, skills, values, beliefs and habits is known as education. Most of the time formal education begins at school. It is indisputable that the foundation for a citizen useful for the country is laid at school. Therefore, this book is compiled with the aim of producing good citizens accepted by the society.

The curriculum undergoes amendments from time to time and the textbooks are compiled maintaining conformity to these changes with the aim of preparing you to face the future world filled with technology and scientific advancements. Accordingly, new textbooks were introduced for grade 1,7 and 11 last year. We are introducing textbooks for grade 2 and 8 this year. This process is to be continued in 2018 as well. I hope this textbook from the Educational Publications Department will help you gain experience to become a perfect citizen.

I would like to extend my gratitude to the panel of writers and editors and the staff of the Educational Publications Department who have contributed in compiling this book

**W.D. Padmini Nalika**
Commissioner General of Educational Publications
Educational Publications Department
Isurupaya
Battaramulla
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# Message of the Board of Compilers

This textbook has been compiled in accordance with the new syllabus to be implemented from 2015.

Textbooks are compiled for students. Therefore, we have made an attempt to compile this textbook in a simple and detailed manner making it possible for you to read and understand it on your own.

We have included descriptions, activities and examples to introduce the subject concepts in an attractive manner and to establish them. Moreover, activities are organized from simple to complex to develop an interest to do them.

We have used the terms related to mathematical concepts in accordance with the glossary of technical terms of mathematics compiled by the Department of Official Languages.

Some subject matter learnt during the earlier grades is necessary to learn the subject content in the grade 11 syllabus. Thus, review exercises are included at the beginning of each chapter to revise previous knowledge. You will be prepared by them for the subject content of grade 11.

In addition, students may use the grade 10 book which you have if you need to recall previuos knoweledge.

You will gain maximum benefit from this textbook by reading the chapters and doing the review exercises of each chapter even before your teacher teaches them in the classroom.

We hope that studying mathematics will be an interesting, joyful and productive experience.

Board of Compilers

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09

# Percentages

**By studying this lesson, you will be able to:**

- compute the loan instalment when the interest is computed on the reducing balance.
- compute the interest rate calculated on the reducing balance given the loan instalment.
- solve problems regarding compound interest.

To review what you have learnt so far on percentages do the following problems.

**Review Exercise**

**1.** Compute the following percentages.

**a.** 12% of 800 rupees. **b.** 8% of 1 Kilometre.

**c.** 2.5% of 1200 g. **d.** 25% of 2.5 Litres.

**2.** A shopkeeper buys a wristwatch for Rs 500 and sells it for Rs 600. Calculate the profit as a percentage (of the cost price).

**3.** A person borrows Rs 8000 at an annual simple interest rate of 6%. How much interest will he pay in a year?

**4.** A person borrows Rs 5000 at an annual simple interest rate of 10%. How much interest will she pay after two years?

**5.** Sunimal takes out a loan of Rs 10 000 that charges a monthly simple interest rate of 2%. What is the total amount that he will re-pay if he wants to settle the entire loan in 3 months?

## Introduction

Our daily household expenses fall into two main categories as capital expenses and recurrent expenses. Recurrent expenses are the expenses we incur on a regular basis and would include our spending on food, clothes, medicine and electricity bills.

Capital expenses are one-time expenses that are not repeated on a regular basis. For instance our spending on purchasing a land, a house, a vehicle, machinery or furniture is a capital expense. Such purchases are generally of significant value and may often require a loan from the place of work or from a financial institution.

Money borrowed as a loan is not paid back in full right away. Rather, it is paid back over a long period of time as monthly partial payments.Usually a loan is expected to be repaid with interest. The part of the loan and the interest that is to be paid every month is referred to as a loan instalment. However, some manufacturers or distributors may sell their merchandise on an interest free installment plan to promote sales.

**Example 1**

A furniture manufacturer sells a Rs 30 000 worth wooden wardrobe on an interest free scheme of 12 monthly payments.What is the amount paid as an instalment?

$$\text{Instalment amount} = \text{Rs } \frac{30\,000}{12}$$

$$= \underline{\underline{\text{Rs } 2\,500}}$$

**Example 2**

A festival advance of Rs 5000 is given to employees of government institutions. This interest free advance has to be paid back in 10 equal monthly instalments. If each installment is deducted from the salary, what is the amount deducted from the salary each month?

$$\text{The amount deducted from the monthly salary each month} = \text{Rs } \frac{5\,000}{10}$$

$$= \underline{\underline{\text{Rs } 500}}$$

## 9.1 Calculating interest on the reducing balance

There are several ways of charging interest on a loan. Calculating the loan interest on the reducing balance is a common method. Let us explore this.

When you borrow money to repay in monthly instalments, or if you purchase an item with a down payment with the understanding that the rest of the money will be paid in instalments, most of the time you will be expected to pay an interest on the loan. Here, the loan is paid in monthly instalments. The interest calculation is based on the outstanding loan balance. That is the balance money that remains in the borrowers hands as the loan is repaid during the loan term. As the borrower repays instalments, the remaining loan balance reduces over time. Interest is then charged only on the loan amount that the borrower still holds. Therefore this method of calculating the interest is called, computing the interest on the reducing balance.

Once the total interest to be paid is calculated, the monthly installments are determined, such that every installment is of the same value.

Study the following examples to understand the method of calculating the loan interest on the reducing balance and the value of an installment.

**Example 1**

Mr Wickramasinghe has borrowed Rs 30 000 as a business loan from a bank that charges an annual interest rate of 24%. The loan has to be paid back in six equal monthly instalments and the interest is calculated on the reducing balance. Calculate the value of a monthly instalment.

The loan amount = Rs 30 000

Amount due from the principal loan per month without interest $= \text{Rs } \frac{30\ 000}{6}$

$= \text{Rs } 5\ 000$

In this method, the outstanding loan balance is reduced by Rs 5000 every month and interest is charged only on the loan balance.

Annual interest rate = 24%

Monthly interest rate = 2%

Interest charged for the first month $= \text{Rs } 30\ 000 \times \frac{2}{100}$

$= \text{Rs } 600$

Interest charged for the second month $= \text{Rs } 25\ 000 \times \frac{2}{100}$

$= \text{Rs } 500$

Interest charged for the third month $= \text{Rs } 20\ 000 \times \frac{2}{100}$

$= \text{Rs } 400$

Interest charged for the fourth month $= \text{Rs } 15\ 000 \times \frac{2}{100}$

$= \text{Rs } 300$

Interest charged for the fifth month $= \text{Rs } 10\ 000 \times \frac{2}{100}$

$= \text{Rs } 200$

Interest charged for the sixth month $= \text{Rs } 5\,000 \times \frac{2}{100}$

$= \text{Rs } 100$

The total interest paid $= \text{Rs } 600 + 500 + 400 + 300 + 200 + 100$

$= \text{Rs } 2\,100$

Total amount to be paid after 6 months = loan without interest + interest

The total amount paid $= \text{Rs } 30\,000 + 2\,100$

$= \text{Rs } 32\,100$

$\therefore$ value of a monthly installment $= \text{Rs } 32\,100 \div 6$

$= \underline{\underline{\text{Rs } 5\,350}}$

Calculating the interest as in the above method could be lengthy and time consuming. Therefore we adopt the following easier method to calculate the interest.

Interest charged on amount due per month from the principal

loan $= \text{Rs } 5\,000 \times \frac{2}{100}$

$= \text{Rs}100$

Therefore,

the total interest paid $= \text{Rs } 100 \times 6 + 100 \times 5 + 100 \times 4 + 100 \times 3 + 100 \times 2 + 100 \times 1$

$= \text{Rs } 100\,(6 + 5 + 4 + 3 + 2 + 1)$

$= \text{Rs } 100 \times 21$

$= \text{Rs } 2\,100$

where 21 is the total number of portions of the loan that is to be paid within the six months. We will call this the number of month units. We calculate the number of month units as

month units $= 6 + 5 + 4 + 3 + 2 + 1 = 21$

This can be thought of as a sum of an arithmetic progression and can be found using the formula $\frac{n}{2}(a + l)$. Then

month units $= \frac{6}{2}(6 + 1)$

$= 3 \times 7$

$= 21$

That is,

$$\text{month units} = \frac{\text{number of instalments}}{2} \times (\text{number of instalments} + 1)$$

**Example 2**

A television priced at Rs 25 000 can be purchased by making a down payment of Rs 7 000 and paying the remainder by 12 equal monthly installments. If an annual interest rate of 18% is charged on the loan,where the interest is calculated on the reducing balance, find the value of a monthly installment.

Price of the television = Rs 25 000
The down payment = Rs 7 000
$\therefore$ Balance to be paid in installments = Rs 25 000 – 7 000
= Rs 18 000
Duration of the loan = 12 month
$\therefore$ Portion of the loan paid in one month = Rs 18 000 $\div$ 12
= Rs 1 500
$\therefore$ Interest paid for a month unit = Rs 1 500 $\times \frac{18}{100} \times \frac{1}{12}$
= Rs 22.50
The number of month units the loan is paid over = $\frac{12}{2}(12+1)$
= $6 \times 13$
= 78
$\therefore$ Total interest paid = Rs 22.50 $\times$ 78
= Rs 1 755
$\therefore$ Total amount paid = Rs 18 000 + 1 755
= Rs 19 755
$\therefore$ The amount of an installment = Rs 19 755 $\div$ 12
= Rs1 646.25

**Example 3**

The following advertisement was displayed in a shop:

> A washing machine worth Rs 30 000 is available for a down payment of Rs 5000 and 10 equal monthly instalments of Rs 2720.

If the interest on the loan was calculated on the reducing loan balance, calculate the interest rate.

Price of the washing machine = Rs 30 000
The down payment = Rs 5 000
Balance to be paid in installments = Rs 30 000 – 5 000
= Rs 25 000
Portion of the loan paid according to the one month = Rs 25 000 $\div$ 10
= Rs 2 500
The amount paid according to the installment plan = Rs 2 720 $\times$ 10
= Rs 27 200
Total interest paid = Rs 27 200 – 25 000
= Rs 2 200
The number of month units the loan is paid over $= \frac{10}{2}(10 + 1)$
$= 55$
Interest for a monthly unit = Rs 2 200 $\div$ 55
= Rs 40
The annual interest rate $= \frac{40}{2\,500} \times 100\% \times 12$
$= \underline{\underline{19.2\%}}$

**Exercise 9.1**

**1.** Sandamini takes a loan of Rs 50 000 from a bank that charges an annual interest rate of 12%. The loan should be repaid in 10 equal monthly instalments.

(i) Find the amount due from the principal loan amount each month.
(ii) Find the interest charged on the principal loan amount each month.
(iii) For how many month units should she pay the interest?
(iv) Find the total interest she should pay under the reducing loan balance
(v) Find the amount of a monthly installment.

**2.** A government servant can acquire a loan up to ten times of his monthly salary at an annual interest rate of 4.2%. The loan has to be repaid within 5 years in equal monthly instalments. If Nimal draws a salary of Rs 30 000 per month; determine the following.

(i) How much can Nimal acquire as a loan?
(ii) What is the duration of the loan in months?
(iii) If the interest is charged on the reducing balance, calculate the total interest due.
(iv) What is the total amount due under the reducing loan balance?
(v) Find the amount of a monthly installment.

**3.** A dining table worth Rs 35 000 can be purchased with a Rs 5000 cash down payment and the rest paid in 15 equal monthly instalments. If the loan is charged an 18% annual interest on the reducing loan balance, calculate the amount of a monthly installment.

**4.** A motor cycle priced at Rs 150 000 for outright purchase can be bought by making a down payment of Rs 30 000 and paying the rest in 2 years in equal monthly installments. If a 24% annual interest rate is charged on the loan, where the interest is calculated on the reducing loan balance, find the amount of a monthly installment.

**5.** Mr Kumar has acquired a loan of Rs 12 000 which he intends to repay in 6 equal monthly instalments. The amount paid in a monthly installment is Rs 2 500.

(i) Find the amount due from the principal loan amount each month.
(ii) Find the total interest he should pay.
(iii) Find the total amount paid in insallments.
(iv) What is the number of month units?
(v) Find the interest for a month unit.
(vi) Find the annual interst rate.

**6.** A refrigerator priced at Rs 36000 for outright purchase can be bought by making a down payment of Rs 6000 and paying the rest in 24 equal monthly installments of Rs 1500. If the interest on the loan is calculated on the reducing loan balance, find the annual interest rate.

**7.** A sewing machine is available at Rs 23 000 for outright purchase. A person going for an installment plan can purchase it by making a down payment of 5 000 and paying the rest in 10 equal monthly installments of Rs 2 000. If the interest on the loan is calculated on the reducing loan balance, find the annual interest rate.

## 9.2 Compound Interest

An alternative way of calculating interest for an amount of money borrowed or deposited is the compound interest method. Let us explore how the interest is calculated under this method through an example.

At a bank that pays an annual interest rate of 10%, the account statement provided at the end of 3 years, to a person who has maintained a Rs 25 000 fixed account is as follows.

| Date | Description | Deposits (Rs) | Interest (Rs) |
|---|---|---|---|
| 2013.01.01 | cash deposit | 25 000.00 | _ |
| 2013.12.31 | interest | _ | 2 500.00 |
| 2014.01.01 | balance | 27 500.00 | _ |
| 2014.12.31 | interest | _ | 2 750.00 |
| 2015.01.01 | balance | 30 250.00 | _ |
| 2015.12.31 | interest | _ | 3 025.00 |
| 2016.01.01 | balance | 33 275. 00 | _ |

According to the above statement the depositor has earned Rs 2 500 as interest in the year 2013. It is clear that the interest earned is 10% of the principal deposit. The sum of the money deposited in 2013 and the interest earned in the year 2013 that amounts to 27 500 is considered as the total in the account on 2014.01.01. Furthermore, the interest earned in the year 2014 is 2750 and it is 10% of the total Rs 27500. It is apparent that the interest earned at the end of the year is added to the account balance and the interest for the next year is calculate on the new balance.

In this manner, when the interest is calculated every year, not only the principal but the previously earned interest also earns interest. The addition of interest to the principal is called compounding and the interest calculation on the compounded amount is called the **compound interest** method.

The compound interest method can be used when calculating interest on a loan as well as on a deposit.

**Example 1**

If a person takes a loan of Rs 10 000 at a compound interest rate of 10% per year, calculate the total amount required to repay the entire loan in two years.

Loan amount = Rs 10 000

Compound interest rate = 10%

Interest for the first year $= \text{Rs } 10\,000 \times \frac{10}{100}$

$= \text{Rs } 1\,000$

Loan amount at the end of the first year = Rs 10 000 + 1 000

= Rs 11 000

Interest for the second year $= \text{Rs } 11\,000 \times \frac{10}{100}$

$= \text{Rs } 1\,100$

Loan amount at the end of the second year = Rs 11 000 + 1 100

= Rs 12 100

As in the above example, compound interest can be calculated separately for each year and added to the loan amount to find the total loan amount due.

**Example 2**

Amal invests Rs 50 000 for 3 years in a fixed deposit account which pays 6% annual interest compounded yearly. Nimal invests Rs 50 000 in an account which pays 6% annual simple interest. Calculate the amounts received by Amal and Nimal at the end of three years separately.

Total amount received by Amal at the end of the first year $= \text{Rs } 50\,000 \times \frac{106}{100}$

$= \text{Rs } 53\,000.00$

Total amount received by Amal at the end of the second year $= \text{Rs } 53\,000 \times \frac{106}{100}$

$= \text{Rs } 56\,180.00$

Total amount received by Amal at the end of the third year $= \text{Rs } 56\,180 \times \frac{106}{100}$

$= \underline{\underline{\text{Rs } 59\,550.80}}$

Interest received by Nimal at the end of the third year $= \text{Rs } 50\,000 \times \frac{6}{100} \times 3$

$= \text{Rs } 9\,000.00$

Total amount received by Nimal at the end of three years $= \text{Rs } 9\,000 + 50\,000$

$= \underline{\underline{\text{Rs } 59\,000.00}}$

Total amount received by Amal at the end of three years can also be obtained by

$\text{Rs } 50\,000 \times \frac{106}{100} \times \frac{106}{100} \times \frac{106}{100}$

$= \text{Rs } 59\,550.80$ .

**Exercise 9.2**

**1.** If a person takes a loan of Rs 5 000 at a compound interest rate of 5% per year, calculate the total amount required to repay the entire loan in two years.

**2.** If a person deposits Rs 6 000 into an account paying 7% annual interest compounded yearly, how much money will be in the account after 2 years?

**3.** Radha deposits Rs 8 000 in an account paying 12% annual interest compounded yearly. After one year the bank interest rate drops to 10%. How much money will Radha receive in total as interest at the end of 2 years?

**4.** Hashan and Caseem are two friends. If Hashan lends Rs 25 000 at a simple annual interest rate of 15% and Caseem lends 25 000 at an interest rate of 14% compounded yearly on the same date, calculate who receives more money after three years.

**5.** At the beginning of a year a person deposits Rs 40 000 in a bank that pays 12% annual interest compounded every six month (semi annually). Compute how much money in total he receives at the end of the year.

6. A person who has loaned a certain amount at 8% interest compounded yearly, receives Rs 432 as compound interest at the end of two years. Find the amount that he loaned.

**Miscellaneous Exercise**

**1.** A television is priced at Rs 45 000. A person who buys it outright for cash, receives a discount of 6% and a person going for an instalment plan can make a down payment of Rs 9 000 and pay the remainder in 12 equal monthly instalments. A 24% annual interest rate is charged on the loan, where the interest is calculated on the reducing balance.

(a) What is the total amount paid if the TV is bought outright for cash?
(b) What is the total amount paid if the TV is bought on an instalment plan?
(c) How profitable is it to buy the TV outright for cash than on an installment plan?

**2.** A person takes a loan of Rs 100 000 at an interest rate of 4.2% compounded annually and deposits it in a bank that pays 8% interest compounded annually. What is the profit of his investment after 2 years?

**3.** A person takes a loan at a certain interest rate compounded annually. If he has to pay Rs 14 400 to settle the entire loan in 2 years or Rs 17 280 to settle it in 3 years, calculate the amount he borrowed and the compound interest rate.

# 10 Share Market

**By studying this lesson you will be able to**

- identify the stock market and its nature
- identify terms related to the stock market
- calculate the dividends gained by investing in the stock market
- solve problems related to shares

## Introduction

In this lesson we will consider businesses in Sri Lanka which are incorporated companies registered under the Companies Act No. 07 of 2007. These companies may be owned by an individual or a group of individuals. Of these companies, the companies which are limited by shares can be classified as follows.

- Private limited companies
- Public limited companies

Public limited companies raise capital to commence or continue their business by issuing shares or debentures to the general public. The public is notified regarding the issue of shares through the media. When the public buy shares, they have the right to sell these shares to others. The stock market is where the trading of these shares occurs.

## Stock Market

The Stock Market (Stock Exchange) is a place where securities such as shares and debentures issued by companies are traded. The Colombo Stock Exchange (CSE) is the organization responsible for the operation of the stock market in Sri Lanka. The CSE is licensed by the Securities and Exchange Commission of Sri Lanka (SEC) to operate as a stock market in Sri Lanka. The SEC regulates and oversees the stock market. Companies are admitted to the official list of the CSE as listed companies. On April 21st 2015 there were 297 companies listed in the Sri Lankan stock market. The public who wish to buy or sell securities in the stock market must register with one or more of the 15 licensed stockbroker firms of the CSE who attend to the transactions. Some brokers offer trading through the internet.

## 10.1 Shares

Public limited companies which are listed in the stock market, that wish to raise capital by involving the public, do this by issuing "shares". A share is one of the equal parts into which a company's stated capital is divided.

When a company issues new shares, the price of a share is decided by the company itself. The public can invest in as many shares as they like. An investor who buys shares in a company becomes a part owner of that company. The stake he has in the company is proportional to the number of shares he owns.

To understand this further, consider the following example.

An investor buys 10 000 shares from the 100 000 shares issued to the public by a certain company. Then the investor has a share of $\frac{10\,000}{100\,000}$ in the company. Let us express this as a percentage.

$$\frac{10\,000}{100\,000} \times 100\% \quad = 10\%.$$

Therefore, the investor has 10% ownership of the company.

**Example 1**

Company $C$ which has a stated capital of Rs 10 000 000, issues 100 000 shares to the public at Rs 100 per share. Vishwa buys 5000 of these shares.

(i) Express Vishwa's share in the company,
  (a) as a fraction
  (b) as a percentage.
(ii) Find the amount that Vishwa invested in company $C$.

(i) Total number of shares issued by the company = 100 000
Number of shares Vishwa bought = 5 000

(a) Vishwa's share in the company as a fraction = $\frac{5\,000}{100000} = \frac{1}{20}$

(*b*) Vishwa's share in the company as a percentage $= \frac{1}{20} \times 100\%$

$= \underline{\underline{5\%}}$

(ii) Price of a share $=$ Rs 100

The number of shares Vishwa purchased $= 5\,000$

The amount Vishwa invested $=$ Rs $100 \times 5\,000$

$= \underline{\underline{\text{Rs } 500\,000}}$

## Dividends

When a listed company issues new shares, it gives notice of the benefits that shareholders will enjoy. Dividends are payments out of the earnings of the company which some companies give their shareholders as a benefit. It is expressed in terms of the amount paid per share. This is paid out quarterly or annually.

For example, a company may pay its shareholders annual dividends of Rs 5 per share. The company has the right to change this amount with time.

Let us consider again the above example to clarify this further.

**Example 1**

Company *C* pays annual dividends of Rs 4 per share, for the Rs 100 shares it issued, of which Vishwa bought 5 000 shares.

(i) Find the annual income that Vishwa receives through this investment.

(ii) Express Vishwa's annual income as a percentage of the amount he invested.

(i) The number of shares Vishwa owns $= 5000$

Annual dividends per share $=$ Rs 4

$\therefore$ Vishwa's annual income $=$ Rs $5000 \times 4$

$= \underline{\underline{\text{Rs } 20\,000}}$

(ii) Amount that Vishwa invested $=$ Rs $100 \times 5\,000$

$=$ Rs $500\,000$

$\therefore$ Vishwa's annual income as a percentage of his investment $= \frac{20\,000}{500\,000} \times 100\%$

$= \underline{\underline{4\,\%}}$

Now do the following exercise pertaining to the facts on initial investments in shares.

## Exercise 10.1

**1.** An investor purchases 1000 shares of value Rs 25 per share in the company Sasiri Apparels.

(i) How much did the person invest?

(ii) If the company pays annual dividends of Rs 4 per share, find the investor's annual dividends income.

**2.** Complete the following tables.

(i)

| Price of a share (Rs) | Number of shares | Amount invested (Rs) |
|---|---|---|
| 10 | 2500 | .............. |
| 20 | 5000 | .............. |
| ....... | 500 | 50 000 |
| ....... | 4000 | 80 000 |
| 30 | .......... | 30 000 |
| 45 | .......... | 135 000 |

(ii)

| Number of shares | Annual dividends per share (Rs) | Annual dividends income (Rs) |
|---|---|---|
| 500 | 2 | .............. |
| 1000 | 3.50 | .............. |
| ....... | 5 | 5000 |
| ....... | 2.50 | 500 000 |
| 2000 | .......... | 8000 |
| 750 | .......... | 2250 |

**3.** A public limited company issues 10 000 000 shares to the public at Rs 25 per share to raise its capital. The company pays annual dividends of Rs 5 per share. Sujeeva purchases 50 000 shares in this company.

(i) Find the stated capital of this company.

(ii) Find the amount Sujeeva invests in this company.

(iii) Find the dividends Sujeeva receives annually through this investment in shares.

(iv) What percent of the amount he invested is the annual dividends he receives?

**4.** Mehela bought a certain number of shares at Rs 20 per share in a company which pays annual dividends of Rs 3 per share. His dividends income at the end of a year from this investment was Rs 12 000.
(i) Find the number of shares Mahela owns in this company.
(ii) Find the amount Mahela invested to buy shares in this company.

**5.** Ganesh spends exactly half of Rs 100 000 to buy a certain number of Rs 25 shares in a company that pays annual dividends of Rs 4 per share. He decides to deposit the remaining amount in a financial institute which pays an annual interest rate of 12%. Show with reasons, which of the two investments is more advantageous.

## 10.2 Trading in the stock market

We learnt earlier that only listed companies are able to trade their shares in the stock market. Let us consider the following note to learn about the trading of shares that is done after a listed company has issued new shares to the public.

Nethmi Limited Company which pays annual dividends of Rs 2 per share, issued 100 000 shares to the public at Rs 10 per share at its initial offering. In a year, the value of these shares had increased in the stock market to Rs 20 per share, at which time Nadeesha bought 1000 shares. A few years later, when the price of a share in this company had increased to Rs 28, Nadeesha sold her 1000 shares.

The **primary market** is where investors buy new shares issued by a company. Shares can only be bought in the primary market, and the purchases are done directly from the issuing company at the initial price stated by the company itself. However, subsequent to the original issuance of shares in the primary market, trading of shares can be done by investors in the **secondary market**. The price of a share in the secondary market is called the **market price**. The market price of a share varies with time depending on the demand.

In the above example, the price of a share in Nethmi Limited Company increased from Rs 10 to Rs 20 in a year and subsequently after several years to Rs 28. Such increases and decreases in the price of a share occur in the secondary market, where investors are able to trade their shares.

## Capital Gain

The price at which a share is bought in a company at the initial offering or later at the market price is called the **purchase price** of the share and the market price at which it is sold is called the **selling price** of the share.

When an investor sells shares he owns he may make a profit or incur a loss.

When he sells his shares,

if the selling price > purchase price, then he makes a capital gain and
the capital gain = selling price – purchase price.

Similarly,

if the selling price < purchase price, then the investor incurs a capital loss and
the capital loss = purchase price – selling price.

**Example 1**

Mr. Perera who invests in the stock market, bought 2000 shares in a certain company when the market price of a share was Rs 20. When the market price of a share increased to Rs 25, he sold all his shares.

(i) Find the amount Mr. Perera invested in the company.
(ii) Find the amount he made by selling the shares.
(iii) Find his capital gain.
(iv) Express his capital gain as a percentage of his investment.

(i) Amount invested in the company = Rs $20 \times 2\,000$
= Rs 40 000

(ii) Amount received by selling the shares = Rs $25 \times 2\,000$
= Rs 50 000

(iii) Capital gain = Rs 50 000 – 40 000
= Rs10 000

(iv) The capital gain as a percentage of the amount invested $= \frac{10\,000}{40\,000} \times 100\%$

$= 25\%$

The capital gain percentage mentioned in (iv) above can be calculated using the price of a share too.

Purchase price of a share = Rs 20
Selling price of a share = Rs 25

$\therefore$ capital gain as a percentage of the amount invested $= \dfrac{25-20}{20} \times 100\%$

$$= \frac{5}{20} \times 100\%$$

$$= \underline{\underline{25\%}}$$

**Example 2**

Mr. Mohamed spent a certain amount from the Rs 96 000 he had in hand, to buy shares at Rs 18 per share, in Company $A$ which pays annual dividends of Rs 2 per share. He spent the remaining amount to buy shares at Rs 21 per share, in Company $B$ which pays annual dividends of Rs 3.50 per share. At the end of a year he received Rs 1000 more as annual dividends income from Company $B$, than the amount he received from Company $A$.

(i) By taking the amount that Mr. Mohamed invested in Company $A$ as $x$, construct an equation in $x$.

(ii) Find the amount he invested in each company by solving the above equation.

(iii) Find the number of shares he had in each company.

(iv) Find the annual dividends income he received from each company.

After receiving the annual income, Mr. Mohamed sold all the shares he owns in both companies at the market price of Rs 20 per share.

(v) Find the total amount he made by selling all his shares in both companies.

(vi) Show that Mr. Mohamed's expectation of making a profit of 20% on his original investments through the dividends income and capital gains was not fulfilled.

(i) Number of shares bought in company $A$ $= \dfrac{x}{18}$

The annual dividends income from company $A$ $=$ Rs $\dfrac{x}{18} \times 2 = \dfrac{x}{9}$

Similarly,

the annual dividends income from company $B$ $=$ Rs $\dfrac{(96\,000 - x)}{21} \times 3.50$

$$= \text{Rs } \frac{(96\,000 - x)}{21} \times \frac{7}{2}$$

$$= \text{Rs } \frac{(96\,000 - x)}{6}$$

$$\therefore \quad \frac{(96\,000 - x)}{6} - \frac{x}{9} = 1000 \text{ is the required equation.}$$

(ii) $$\frac{(96\,000 - x)}{6} - \frac{x}{9} = 1000$$

$$18 \times \frac{(96\,000 - x)}{6} - 18 \times \frac{x}{9} = 18 \times 1000$$

$$3\,(96\,000 - x\,) - 2x = 18\,000$$
$$288\,000 - 3x - 2x = 18\,000$$
$$288\,000 - 18\,000 = 5x$$
$$270\,000 = 5x$$
$$x = 54\,000$$

$\therefore$ amount invested in company $A$ is Rs 54 000.

Amount invested in company $B$ = Rs 96 000 – 54 000 = Rs 42 000

(iii) Number of shares owned in company $A$ $= \frac{54\,000}{18} = 3000$

Number of shares owned in company $B$ $= \frac{42\,000}{21} = 2000$

(iv) Income received from investment in company $A$ = Rs $3000 \times 2$ = Rs 6000

Income received from investment in company $B$ = Rs $2000 \times 3.50$ = Rs 7000

(v) Amount received by selling shares in company $A$ = Rs $3000 \times 20$ = 60 000

Amount received by selling shares in company $B$ = Rs $2000 \times 20$ = 40 000

$\therefore$ total amount received by selling the shares in both the companies = Rs 60 000 + 40 000

= Rs 100 000

The annual dividends income received from both companies = Rs 6000 + 7000

= Rs 13 000

$\therefore$ the sum of the amounts received as dividends income and by selling the shares } = Rs 100 000 + 13 000

= Rs 113 000

Amount invested to buy shares in the two companies $= \text{Rs } 96\,000$

Profit $= \text{Rs } 113\,000 - 96\,000$
$= \text{Rs } 17\,000$

$\therefore$ the profit from the investment as a percentage of the amount invested $\Big\} = \dfrac{17\,000}{96\,000} \times 100\%$

$= 17.7\%$

Since 17.7% < 20%, Mr. Mohamed's expectations were not fulfilled.

**Exercise 10.2**

**1.** Complete the following table.

| Amount invested (Rs) | Market price of a share (Rs) | Number of shares | Income from annual dividends of Rs 3 per share (Rs) |
|---|---|---|---|
| 50 000 | 25 | ................. | .................. |
| ............ | 40 | ................. | 1500 |
| 75 000 | ....... | ................. | 3000 |
| ............. | 15 | 500 | .................. |
| 120 000 | ......... | 2000 | .................. |

**2.** Tharindu invested Rs 60 000 and bought shares in a company at the market price of Rs 30 per share. The company pays annual dividends of Rs 4 per share.

(i) Find the number of shares Tharindu bought.
(ii) Find the annual dividends income that Tharindu receives from this investment.
(iii) What percentage of the amount invested is the annual dividends income?

**3.** Ramesh bought 5000 shares in a certain company when the market price was Rs 40 per share. He then sold all these shares when the market price had increased to Rs 50 per share.

(i) Find Ramesh's capital gain per share.
(ii) Find his capital gain due to selling all the shares.
(iii) Express his capital gain as a percentage of the amount invested.

**4.** A businessman invested Rs 40 000 and bought shares in a certain company at the market price of Rs 40 per share. At the end of a year, he received dividends of 10% on his investment. After receiving this income he sold all his shares at Rs 50 per share.

(i) Find the annual income the businessman received from the company.
(ii) Find the annual dividends the company paid per share.
(iii) Find the amount the businessman received by selling his shares.
(iv) Find his capital gain.
(v) Express his capital gain as a percentage of the amount invested.

**5.** A person who invested in a company and bought shares at the market price of Rs 20 per share, sold all his shares on an occasion when the market price increased. His capital gain from this sale was 80% of his investment.

(i) What was his capital gain per share?
(ii) At what price did he sell each share?

**6.** A person bought shares in a company at the market price of Rs 24 per share and sold the shares when the market price per share was Rs 30. Express his capital gain as a percentage of the amount invested.

**7.** A person bought 1000 shares in a company which pays annual dividends of Rs 6 per share, at the market price of Rs 40 per share. After receiving dividends for a year, he sold his shares on an occasion when the market price of the shares had increased. His total income from the dividends and the sale of the shares was Rs 71 000.

(i) How much was the annual dividends income from this investment?
(ii) What was the selling price of a share?
(iii) Find his capital gain.

**8.** Devinda invested equal amounts in two companies. He bought shares in one company which pays annual dividends of Rs 4 per share, at the market price of Rs 20 per share, and shares in the second company which pays annual dividends of Rs 5 per share, at the market price of Rs 25 per share. Express his income from each company as a percentage of the amount invested. (Hint: Take the amount that he invested in each company to be Rs $x$)

**9.** An investor invested a certain amount from the Rs 70 000 in hand to buy shares in a company which pays annual dividends of Rs 3 per share, at the market price of Rs 30 per share. The rest of the money he invested in a company that pays annual dividends of Rs 4 per share, and bought shares at the market price of Rs 20 per share. If his dividends income for a year from these investments was Rs 9 500, find the amount he invested in each company.

**10.** An investor, who owned 4000 shares in a company which pays annual dividends of Rs 5 per share, sold all his shares when the market price was Rs 45 per share. He spent all the money he received by selling these shares, to buy shares in a company at the market price of Rs 25 per share. From this investment he gained an annual dividends income which was Rs 8 800 more than what he received from his previous investment. Find the annual dividends per share that the second company paid.

## Miscellaneous Exercise

**1.** Malki placed Rs 50 000 for a year in a fixed deposit, with a financial institute which pays an annual interest rate of 12%. At the end of the year she withdrew the money and used the principal amount and the interest she received to buy shares in a company that pays annual dividends of Rs 4 per share. She bought the shares at the market price of Rs 28 per share.

(i) Find the annual interest Malki received from her fixed deposit.
(ii) Find the amount she invested in shares.
(iii) Find the annual dividends income she received from her investment.
(iv) With reasons state whether it would have been more profitable for Malki to have re-invested the principal amount together with the interest in a fixed deposit with the financial institute for another year, than to have invested in shares.

**2.** An investor, who owned 1500 shares in a company which pays annual dividends of Rs 2 per share, sold these shares at the market price of Rs 32 per share after receiving the annual dividends income. He invested the money he received by selling the shares, in another company which pays annual dividends of Rs 2 per share. The market price at which he bought the shares in the second company was Rs 40 per share. Show that the ratio of the dividends income from the first company to the dividends income from the second company is 5:4.

**3.** Udesh took a loan of Rs 40 000 from a financial institute, at an annual simple interest rate of 12%. With this loan, he bought shares at Rs 20 per share, in a company which pays annual dividends of Rs 4.50 per share. After three years, he sold all the shares at the current market price of Rs 28 per share and paid off the loan together with the interest. Show that Udesh made a profit of Rs 28 600 from his investment.

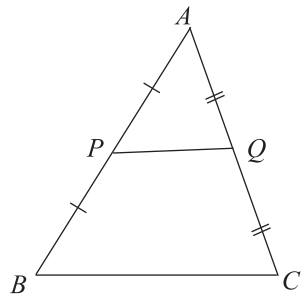
**4.** Upul invests in a company by buying shares when the market price of a share is Rs 48. He plans to sell the shares when the market price has increased sufficiently, so that his capital gain upon selling the shares will be 30% of his investment. At what price should he sell a share, for him to achieve this?

# 11 The Midpoint Theorem

**By studying this lesson you will be able to**
- understand the midpoint theorem and its converse,
- perform calculations and prove riders using the midpoint theorem and its converse.

## 11.1 The Midpoint Theorem

The midpoint theorem is a result related to the lengths of the sides of a triangle. Let $P$ be the midpoint of the side $AB$, and $Q$ be the midpoint of the side $AC$ of the triangle $ABC$.




Then,

$AP = PB$ and $AQ = QC$.

This can also be written as $AP = PB = \frac{1}{2}AB$ and $AQ = QC = \frac{1}{2}AC$.

$PQ$ is the line segment obtained by joining the midpoints of the sides $AB$ and $AC$.

**Theorem**
The straight line segment through the midpoints of two sides of a triangle is parallel to the third side and equal in length to half of it.

In relation to the above figure, according to the theorem,

$PQ /\!/ BC$ and

$PQ = \frac{1}{2}BC$.

Let us do the following activity to establish this theorem.

**Activity 1**

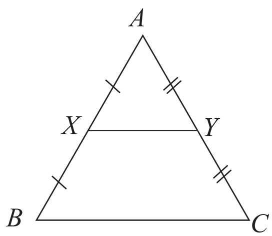
Construct the triangle $ABC$ such that $AB = 6$ cm, $BC = 7$ cm and $CA = 8$ cm. Name the midpoints of $AB$ and $AC$ as $P$ and $Q$ respectively. Join $PQ$

(i) Measure the length of $PQ$ and establish the fact that it is half the length of $BC$.
(ii) Check using a set square or by some other method that $PQ$ is parallel to $BC$.

By doing the above activity, you would have seen that $PQ = \frac{1}{2} BC$ and $PQ//BC$.

Let us consider through an example how the lengths of the sides of rectilinear plane figures related to triangles are found using the midpoint theorem.

**Example 1**




An equilateral triangle $ABC$ of side length 12 cm is represented in the above figure.The midpoints of $AB$ and $AC$ are $X$ and $Y$ respectively.

Determine the following.

(i) The length of $XY$.
(ii) The perimeter of the quadrilateral $BCYX$.

(i) According to the midpoint theorem,

$XY//BC$ and $XY = \frac{1}{2} BC$.

$$\therefore XY = \frac{1}{2} \times 12$$
$$= 6$$

$\therefore$ the length of $XY$ is 6 cm.

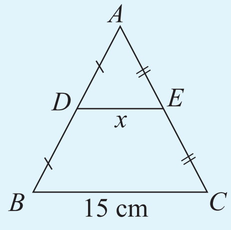
(ii) The perimeter of $BCYX = BC + CY + XY + XB$

$$= 12 + 6 + 6 + 6$$
$$= 30$$

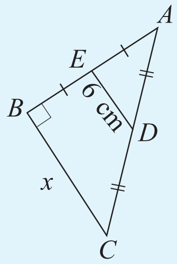
$\therefore$ the perimeter of $BCYX$ is 30 cm.

Exercise 11.1

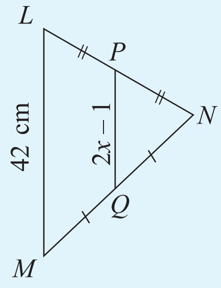
**1.** Determine the value of $x$ in each figure.




(i)

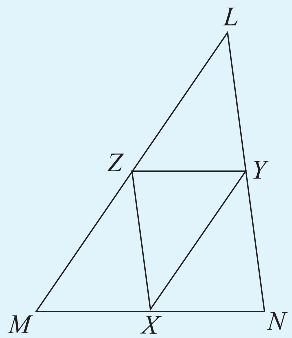



(ii)




(iii)

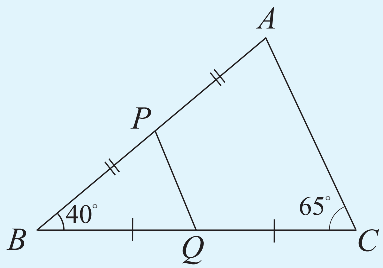
**2.**




In the given figure, $X$, $Y$ and $Z$ are the midpoints of the sides $MN$, $NL$ and $LM$ respectively of the triangle $LMN$. If $MN = 8$ cm, $NL = 10$ cm and $LM = 12$ cm, find the perimeter of the triangle $XYZ$.

**3.** In the quadrilateral $ABCD$, $AC = 15$ cm and $BD = 10$ cm. Find the perimeter of the quadrilateral that is obtained by joining the midpoints of the sides $AB$, $BC$, $CD$ and $DA$.

**4.**

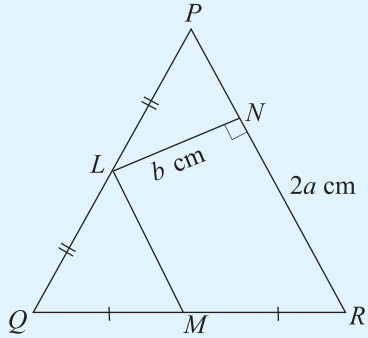



Based on the information in the figure,

(i) if the perimeter of $ABC$ is 22 cm, $AB = 8$ cm and $BC = 10$ cm, find the perimeter of the triangle $PBQ$.

(ii) if $\hat{B} = 40^\circ$, and $\hat{C} = 65^\circ$, find the remaining angles in the quadrilateral $PQCA$.

**5.**



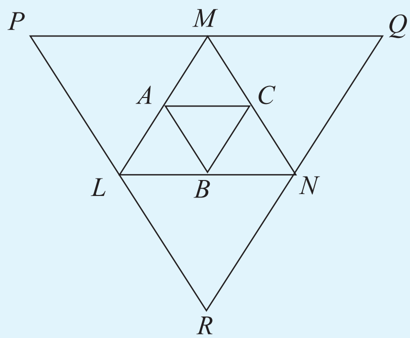

The midpoints of the sides $QR$ and $QP$ of the triangle $PQR$ in the figure are $M$ and $L$ respectively.

$QR + QP = 16$ cm, $PR = 2a$ cm, $LN = b$ cm and $L\hat{N}R = 90^\circ$.

(i) Find the perimeter of the quadrilateral $LMRP$ in term of $a$.

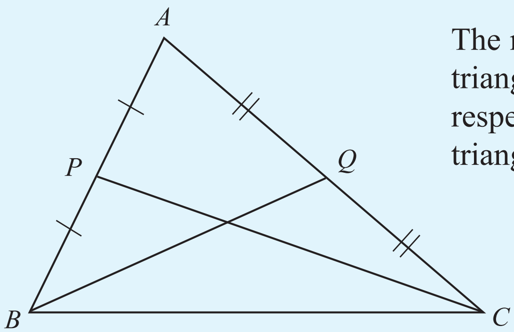
(ii) Express the area of $LMRP$ in terms of $a$ and $b$.

**6.**



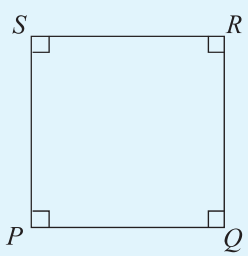

The triangle $LMN$ has been formed by joining the midpoints $L$, $M$ and $N$ of the sides $PR$, $PQ$ and $QR$ respectively of the triangle $PQR$. The triangle $ABC$ has been formed by joining the midpoints $A$, $B$ and $C$ of the sides $LM$, $LN$ and $MN$ of the triangle $LMN$. If the perimeter of the triangle $PQR$ is 12 cm, find the perimeter of the triangle $ABC$.

**7.**




The midpoints of the sides $AB$ and $AC$ of the triangle $ABC$ in the figure are $P$ and $Q$ respectively. Show that the areas of the triangles $PBC$ and $BQC$ are equal.

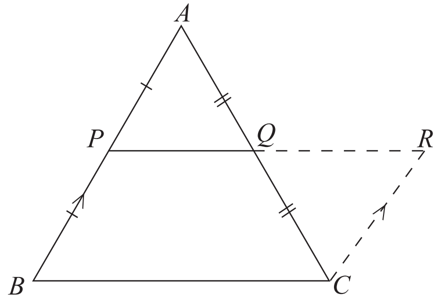
**8.**




The perimeter of the square $PQRS$ in the figure is 60 cm. Find the perimeter of the quadrilateral that is formed by joining the midpoints of the sides of the square and express it in surd form.

## 11.2 Proof of the midpoint theorem

Now let us consider the formal proof of the midpoint theorem.




**Data:** The midpoints of the sides $AB$ and $AC$ of the triangle $ABC$ are $P$ and $Q$ respectively.

**To be proved:**

$PQ//BC$ and

$PQ = \frac{1}{2}BC.$

**Construction:** Draw a straight line through $C$ parallel to $BP$ such that it meets $PQ$ produced at $R$.

**Proof:**

In the two triangles $APQ$ and $QCR$,

$AQ = QC$ (since $Q$ is the midpoint of $AC$)

$A\hat{P}Q = Q\hat{R}C$ (since $AP//RC$, alternate angles)

$A\hat{Q}P = R\hat{Q}C$ (vertically opposite angles)

$\therefore \Delta APQ \equiv \Delta QCR$ (AAS)

$\therefore AP = RC$ and $PQ = QR$ (corresponding sides of congruent triangles)

However, $AP = PB$

$\therefore PB = RC$

In the quadrilateral $BCRP$, $PB = RC$ and $PB//RC$.

$\therefore BCRP$ is a parallelogram.

$\therefore PR = BC$ and $PR//BC$.

However, $PQ = QR$.

$\therefore PQ = \frac{1}{2}PR$

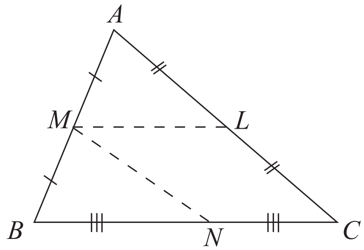
$= \frac{1}{2}BC$ (since $PR = BC$)

$\therefore PQ//BC$ and $PQ = \frac{1}{2}BC.$

Now let us consider how riders are proved using the midpoint theorem.

**Example 1**

*M, N* and *L* are the midpoints of the sides *AB, BC* and *CA* respectively of the triangle *ABC*. Prove that *NCLM* is a parallelogram.




According to the midpoint theorem, $ML = \frac{1}{2} BC$

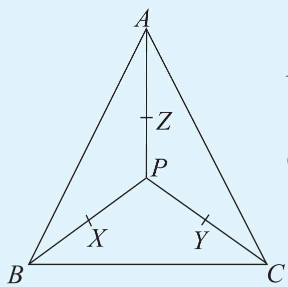
$= NC$ (since *N* is the midpoint of *BC*)

Further, $ML // BC$.

Therefore, a pair of opposite sides of the quadrilateral *NCLM*, are equal and parallel. Therefore, *NCLM* is a parallelogram.

**Exercise 11.2**

**1.**

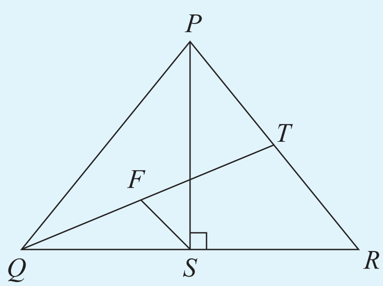



*P* is a point inside the triangle *ABC*. The midpoints of *AP, BP* and *CP* are *Z, X* and *Y* respectively.

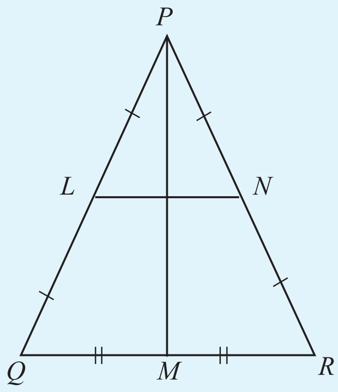
(i) Show that $B\hat{A}C = X\hat{Z}Y$, $A\hat{C}B = Z\hat{Y}X$ and $C\hat{B}A = Y\hat{X}Z$,

(ii) Show that the perimeter of triangle *ABC* is twice the perimeter of triangle *XYZ*.

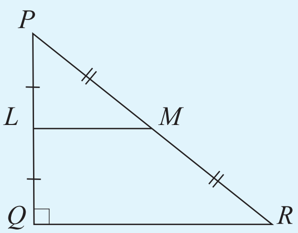
**2.**




The bisector of the angle $Q\hat{P}R$ of the triangle *PQR* in the figure, meets the side *QR* at the point *S*, such that $PS \perp QR$. The midpoint of *QT* is *F*. Prove that $FS // TR$.

**3.** 

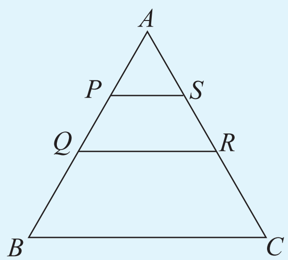

Based on the information in the figure prove that $PM \perp LN$.

**4.** 


Based on the information in the figure prove that

(i) $\Delta PLM \equiv \Delta QLM$.

(ii) area of the quadrilateral $LQRM = \frac{3}{4}$ area of $\Delta PQR$.

**5.** 


The mid points of the sides $AB$ and $AC$ of the triangle $ABC$ are $Q$ and $R$ respectively and the mid points of the sides $AQ$ and $AR$ of the triangle $AQR$ are $P$ and $S$ respectively Prove that $4\,PS = BC$

**6.** (i) Prove that the quadrilateral that is formed by joining the midpoints of the sides of a quadrilateral is a parallelogram.

(ii) Prove that the quadrilateral that is formed by joining the midpoints of the sides of a rectangle is a rhombus.

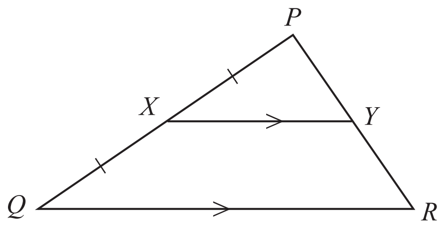
(iii) Prove that the quadrilateral that is formed by joining the midpoints of the sides of a square is a square.

(iv) Prove that the quadrilateral that is formed by joining the midpoints of the sides of a rhombus is a rectangle.

## 11.3 Converse of the Midpoint Theorem

Now let us consider the converse of the midpoint theorem.

**Theorem:**
The straight line through the midpoint of one side of a triangle and parallel to another side, bisects the third side.




$X$ is the midpoint of the side $PQ$ of the triangle $PQR$ in the figure (that is, $PX = XQ$). If $XY//QR$, according to the converse of the midpoint theorem, $Y$ is the midpoint of $PR$. That is, $PY = YR$.

Do the following activity to establish this theorem.

**Activity 2**

- Construct the triangle $PQR$ such that $PQ = 5$ cm, $QR = 6$ cm and $RP = 7$ cm.
- Mark the midpoint of the side $PQ$ as $X$.
- Name the point at which the straight line through $X$ parallel to $QR$ meets the side $PR$ as $Y$.
- Measure the lengths of $PY$ and $YR$ and write down the relationship between these two lengths.
- Similarly, if the straight line through $X$ parallel to $PR$ meets the side $QR$ at $Z$, write down the relationship between the lengths of $QZ$ and $ZR$.

You would have observed by doing the above activity that $PY = YR$ and $QZ = ZR$. This establishes the fact that the straight line through the midpoint of one side of a triangle and parallel to another side, bisects the third side.

Now let us consider some applications of the converse of the midpoint theorem.

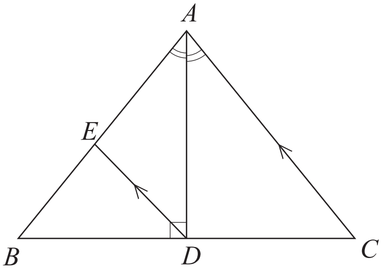
**Example 1**

The bisector of the angle $B\hat{A}C$ of the triangle $ABC$ meets the side $BC$ at $D$. $A\hat{D}B = 90^\circ$. The straight line through $D$ parallel to $CA$ meets the side $AB$ at $E$.

Prove that

(i) $\Delta\ ADB \equiv \Delta\ ADC$,

(ii) $BE = EA$.




(i) In the triangles $ADB$ and $ADC$,

$B\hat{A}D = C\hat{A}D$ (since $AD$ is the bisector of $B\hat{A}C$ )

$AD$ is the common side

$A\hat{D}B = A\hat{D}C$ ($AD \perp BC$)

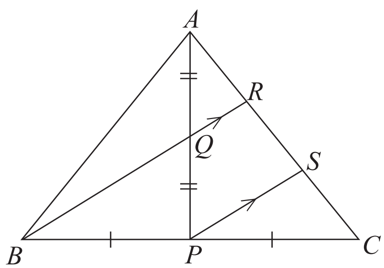
$\therefore \Delta ABD \equiv \Delta ADC$ (AAS)

(ii) $BD = DC$ (corresponding sides of the congruent triangles $ADB$ and $ADC$)

Since $BD = DC$ and $AC // DE$, by the converse of the midpoint theorem,

$BE = EA$.

**Example 2**




$P$ is the midpoint of the side $BC$ of the triangle $ABC$ in the figure. The midpoint of $AP$ is $Q$. $BQ$ produced meets the side $AC$ at $R$. The line through $P$ parallel to $BR$ meets $AC$ at $S$. If $AC = 15$ cm, find the length of $AS$.

In the triangle $APS$, $AQ = QP$ and $QR // PS$.

Therefore, according to the converse of the midpoint theorem,

$AR = RS$ ——— ①

In the triangle $BRC$, $BP = PC$ and $BR // PS$.

Therefore, according to the converse of the midpoint theorem,

$RS = SC$ ——— ②

From ① and ② we obtain, $AR = RS = SC$.

$$\therefore\ AS = \frac{2}{3} AC$$

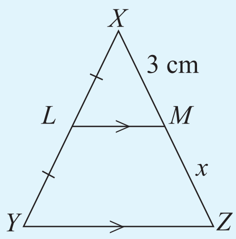
$$= \frac{2}{3} \times 15$$

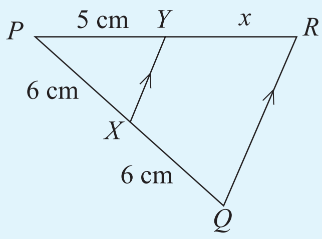
$$= 10.$$

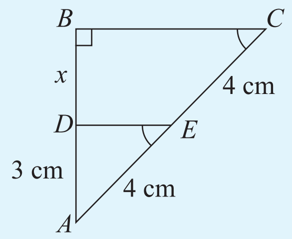
Therefore, the length of $AS$ is 10 cm.

## Exercise 11.3

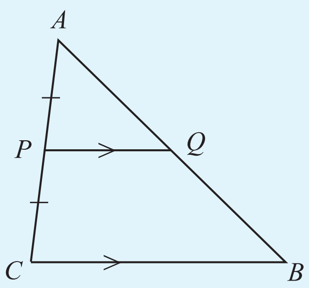
**1.** Find the value of $x$ in each figure.



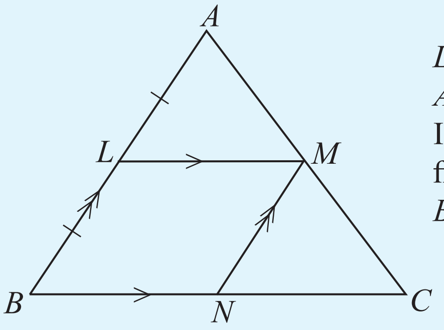

**2.**




The midpoint of the side $AC$ of the triangle $ABC$ is $P$.
$BC = 12$ cm, $AB = 15$ cm and $PQ//CB$.
Find

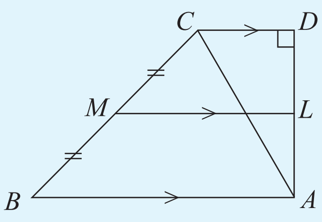
(i) the length of $QB$.
(ii) the length of $PQ$.

**3.**



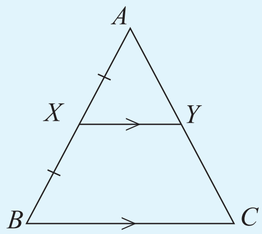

$L$ is the midpoint of the side $AB$ of the triangle $ABC$ in the figure. $LM//BC$ and $MN//AB$. If $AB = 10$ cm, $AM = 7$cm and $BC = 12$ cm, find the length of $MC$ and the perimeter of $BNML$.

**4.**

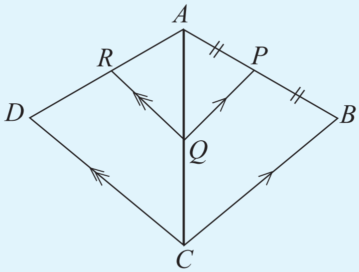



Based on the information in the figure and if $AC = 10$ cm, $AD = 8$ cm and $ML = 10$ cm,

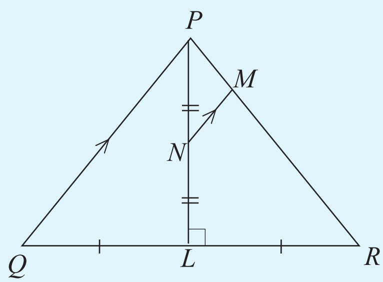
(i) find the length of $DC$.
(ii) find the area of the trapezium $ABCD$.

**5.** 

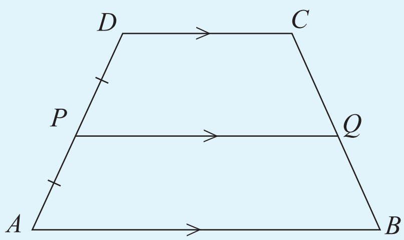

The perimeter of the equilateral triangle *ABC* in the figure is 30 cm. Based on the information in the figure find the perimeter of the trapezium *BCYX*.

**6.** 

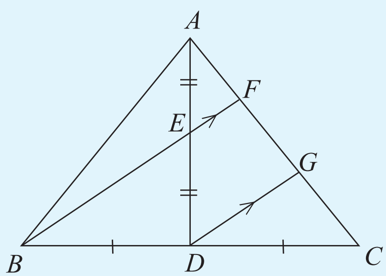

*ABC* and *ADC* in the figure are equilateral triangles and *AB* = 20 cm. Based on the information in the figure, find the perimeter of the region *PQRDCB*.

**7.** 


Find the length of *MN* based on the information in the figure, if *PQ* = 20 cm.

**8.** 

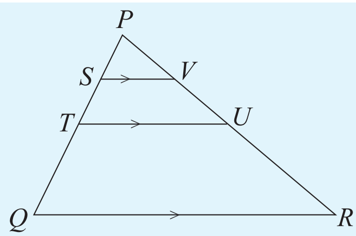

Based on the information in the figure, express the length of *PQ* in terms of the lengths of *AB* and *DC*.

**9.** 


The length of a side of the equilateral triangle *ABC* is *x* cm. If *EF* = *y* cm, based on the information in the figure, express the following in terms of *x* and *y*.

(i) The perimeter of *EDGF*.
(ii) The perimeter of *BDGF*.
(iii) The perimeter of *BDGA*.

**10.**

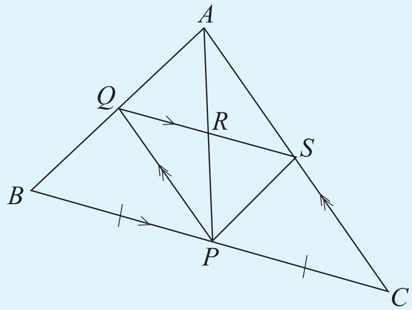



The midpoint of the side $PQ$ of the triangle $PQR$ in the figure is $T$. $S$ is the midpoint of $PT$. The straight lines through $S$ and $T$ drawn parallel to $QR$, meet the side $PR$ at $V$ and $U$ respectively.

(i) Prove that $PV = \frac{1}{4}PR$.

(ii) Determine the ratio $SV : QR$.

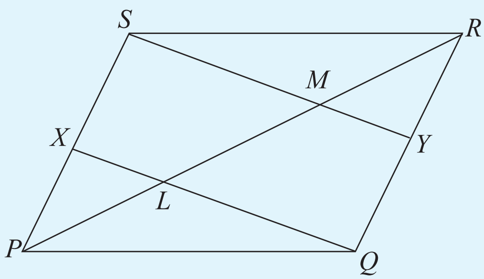
**11.**




Based on the information in the figure prove that $AR = RP$ and PS // $BQ$.

## Miscellaneous Exercise

**1.**

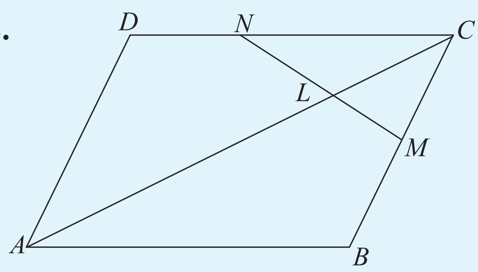



The midpoints of the sides $PS$ and $QR$ of the parallelogram $PQRS$ are $X$ and $Y$ respectively. The lines $XQ$ and $SY$ meet the diagonal $PR$ at $L$ and $M$ respectively.

(i) Prove that $XQYS$ is a parallelogram.

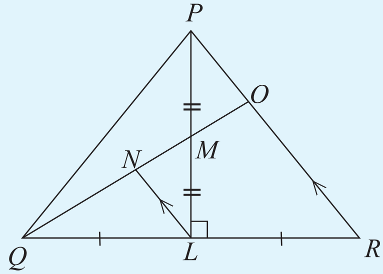
(ii) Prove that $PM = \frac{2}{3}PR$.

**2.**




The midpoints of the sides $BC$ and $CD$ of the parallelogram $ABCD$ are $M$ and $N$ respectively. The straight line $MN$ intersects the diagonal $AC$ at $L$.

Prove that $LC = \frac{1}{4}AC$.

**3.** 


Based on the information in the figure, prove the following.

(i) $QN = NO$.

(ii) $PNLO$ is a parallelogram.

(iii) $\Delta\, POM \equiv \Delta\, NLM$.

(iv) $MO = \frac{1}{4}\, QO$.

**4.** *PQRS* is a parallelogram. Its diagonals intersect at *O*. The midpoint of the side *PQ* is *L*. The midpoint of *LO* is *T*. *PT* produced meets *QR* at *Y*. Prove that,

(i) $PT = TY$,

(ii) *PLYO* is a parallelogram,

(iii) $4\,LT = QR$.

**5.** *Y* and *X* are the midpoints of the sides *PQ* and *PR* respectively of the triangle *PQR*. The lines *QX* and *YR* intersect each other at *L*. The straight line through *Q* parallel to *YR* meets *PL* produced at *M*. The lines *LM* and *QR* intersect at *N*.

(i) Prove that $PL = LM$.

(ii) Prove that $MR // QX$.

(iii) Prove that *QMRL* is a parallelogram.

(iv) Determine $\frac{PL}{PN}$.

# 12 Graphs

**By studying this lesson you will be able to**

- find the solution to a pair of simultaneous equations graphically,
- sketch the graphs of quadratic functions of the form $y = ax^2 + bx + c$,
- analyze the behaviour of a function by considering its graph.

Do the following exercise to recall the facts that you have learnt earlier about straight lines and their graphs.

**Review Exercise**

**1.** ***a.*** On the same coordinate plane, draw the graphs of the following straight lines by calculating the $y$ values corresponding to three selected values of $x$.

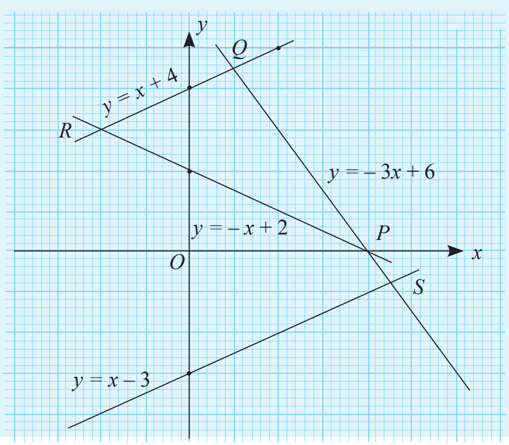
(i) $y = x + 1$ (ii) $y - x = 5$ (iii) $2y = -x - 4$ (iv) $3x + 2y = 6$

***b.*** Write down the coordinates of the points at which each straight line intersects the main axes.

**2.** For each of the straight lines given below, determine whether the given pairs of coordinates lie on it or not.

(i) $y = 2x - 3$ ; (1, 1), (0, 3), (2, 1) (ii) $y = 2x - 3$ ; (0, – 3), $(\frac{1}{2}, 4)$, (1, 3)

**3.** The graphs of four straight lines on a coordinate plane are given below. From the seven given pairs of coordinates, select the ones which correspond to the points *P, Q, R* and *S*, which are the intersecting points of the graphs. Give reasons for your answers.




(– 3, 5), (– 1, 3), (– 1, – 3)

$(\frac{1}{2}, 4\frac{1}{2})$, (2, 0), $(-\frac{5}{2}, \frac{3}{2})$,

$(2\frac{1}{4}, -\frac{3}{4})$

## 12.1 Finding the solution to a pair of simultaneous equations graphically

You have learnt in previous grades how to find the solution to a pair of simultaneous equations using algebraic methods. We will now consider how the solution to a pair of simultaneous equations is found by representing the equations graphically.

Let us consider the following pair of simultaneous equations.

$$y - x = -3$$
$$y + 3x = 5$$

Let us first solve this pair of simultaneous equations algebraically.

$$y - x = -3 \text{ ——— ①}$$
$$y + 3x = 5 \text{ ——— ②}$$

② – ① gives us,

$$(y + 3x) - (y - x) = 5 - (-3)$$
$$\therefore y + 3x - y + x = 5 + 3$$
$$\therefore 4x = 8$$
$$\therefore x = 2$$

By substituting $x = 2$ in ① we obtain,

$$y - x = -3$$
$$\therefore y = -3 + 2$$
$$\therefore y = -1$$

$\therefore$ the solution is

$x = 2$ and $y = -1$.

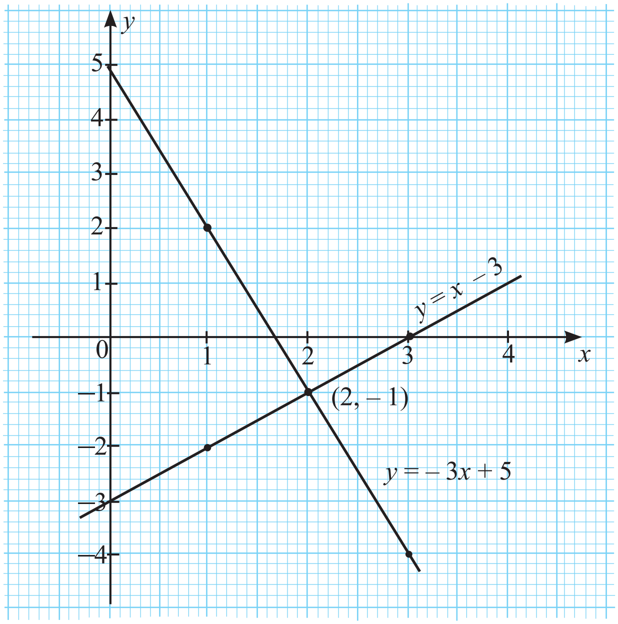
When these two equations are considered separately, they can be written as $y = x - 3$ and $y = -3x + 5$ by making $y$ the subject. Let us first draw the graphs of these straight lines on the same coordinate plane. Two tables of values corresponding to these equations are given below.

$y = x - 3$

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $y$ | $-2$ | $-1$ | 0 |

$y = -3x + 5$

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $y$ | 2 | $-1$ | $-4$ |




The graphs of the straight lines drawn on the same coordinate plane using the above tables of values intersect each other at the point (2, – 1). When the $x$ and $y$ values corresponding to this point are substituted into the given pair of equations, it can be observed that the two sides of the equations are equal. That is, the coordinates $x = 2$ and $y = -1$ of the intersection point of these graphs is the solution to the given pair of simultaneous equations.

This geometric solution to the pair of simultaneous equations is further established by the fact that the same solution was obtained algebraically too.

Accordingly, to find the solution to a pair of simultaneous equations graphically, the straight lines corresponding to the pair of equations should be drawn on the same coordinate plane, and the coordinates of the point of intersection of the graphs should be found. Then the $x$ and $y$ values of the solution are obtained from the $x$-coordinate and the $y$-coordinate respectively of the point of intersection.

In the following example, the method of constructing a pair of simultaneous equations and solving it graphically is considered.

**Example 1**

A person buys 10 stamps, some of value Rs 10 and the rest of value Rs 20. The total value of the stamps is Rs 120.

(i) By taking the number of stamps of value Rs 10 that he bought as $x$ and the number of stamps of value Rs 20 as $y$, construct a pair of simultaneous equations.

(ii) By solving the pair of simultaneous equations graphically, find the number of Rs 10 stamps and the number of Rs 20 stamps he bought.

The relevant pair of simultaneous equations is as follows.

$$x + y = 10 \quad \text{——} ①$$
$$10x + 20y = 120 \quad \text{——} ②$$

Let us represent each of the equations graphically.

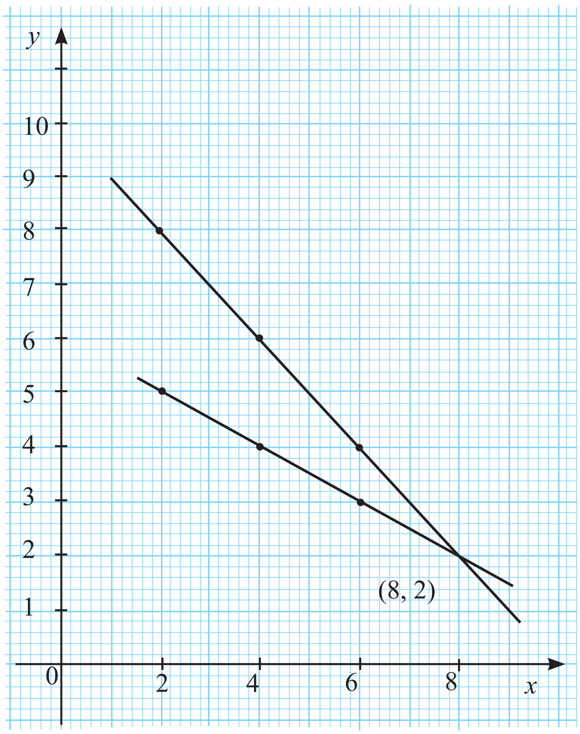
$x + y = 10$; that is, $y = -x + 10$

| $x$ | 2 | 4 | 6 |
|---|---|---|---|
| $y$ | 8 | 6 | 4 |

$10x + 20y = 120$; that is, $y = -\frac{1}{2}x + 6$

| $x$ | 2 | 4 | 6 |
|---|---|---|---|
| $y$ | 5 | 4 | 3 |

We obtain the following pair of straight lines.




When the pair of equations $x + y = 10$ and $10x + 20y = 120$ are represented graphically, they intersect each other at the point (8, 2). Therefore, the solution to the pair of equations is $x = 8$ and $y = 2$. That is, the person bought eight Rs 10 stamps and two Rs 20 stamps.

**Exercise 12.1**

**1.** Solve each of the following pairs of simultaneous equations graphically. Verify your answers by solving the equations algebraically too.

**a.** $y - x = 4$
$y - 2x = 3$

**b.** $y = -2x - 2$
$-2y = -x - 6$

**c.** $3x - 4y = 7$
$5x + 2y = 3$

**2.** A certain school has two Grade 11 classes *A* and *B*. If five students from class *A* move to class *B*, then the number of students in class *B* will be twice the number of students in class *A*. However, if five students from class *B* move to class *A*, then the number of students in the two classes will be equal.

(i) Construct a pair of simultaneous equations by taking the number of students initially in class *A* as *x* and the number of students initially in class *B* as *y*.

(ii) Draw the graphs of the pair of equations on the same coordinate plane and hence find the number of students in class *A* and in class *B* separately.

## The graphs of quadratic functions

Do the following exercise to recall what you have learnt in Grade 10 regarding the graphs of quadratic functions of the form $y = ax^2$ and $y = ax^2 + b$.

**Review Exercise**

**1.** An incomplete table of $x$ and $y$ values prepared to sketch the graph of the function $y = x^2 - 5$ is given below.

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | $4$ | ___ | $-4$ | $-5$ | ___ | $-1$ | $4$ |

**a.** (i) Complete the table.
(ii) Sketch the graph of the above function by selecting a suitable scale.

**b.** Using your graph, write down
(i) the minimum value of the function.
(ii) the coordinates of the minimum point.
(iii) the interval of values of $x$ for which the function is negative.
(iv) the interval of values of $x$ for which the function is increasing positively.
(v) the value of $x$ when $y = -1$.

**2.** (i) Complete the following table to sketch the graph of the function $y = -2x^2 + 4$.

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | $-14$ | ___ | $2$ | $4$ | $2$ | $-4$ | $-14$ |

(ii) Sketch the graph of the function by selecting a suitable scale.

Using the graph,

(iii) write down the coordinates of the turning point of the graph.
(iv) find the $x$ values for which the function takes the value 0.
(v) write down the interval of values of $x$ for which the function is decreasing negatively.
(vi) Write down the interval of values of $x$ for which $y \leq 2$.
(vii) Obtain an approximate value for $\sqrt{2}$, to the first decimal place.

**3.** Complete the following table without drawing the graphs of the given functions.

| Function | Nature of the turning point (maximum/ minimum) | Equation of the axis of symmetry | Maximum/ minimum value | Coordinates of the turning point |
|---|---|---|---|---|
| (i) $y = 2x^2$ | ........................ | ........................ | ........................ | ........................ |
| (ii) $y = \frac{1}{2}x^2$ | ........................ | ........................ | ........................ | ........................ |
| (iii) $y = x^2 + 3$ | ........................ | ........................ | ........................ | ........................ |
| (iv) $y = 1 - 2x^2$ | Maximum | $x = 0$ | 1 | (0, 1) |
| (v) $y = -3x^2 - 4$ | ........................ | ........................ | ........................ | ........................ |
| (vi) $y = \frac{3}{2}x^2 - 2$ | ........................ | ........................ | ........................ | ........................ |

## 12.2 The graph of a function of the form $y = ax^2 + bx + c$

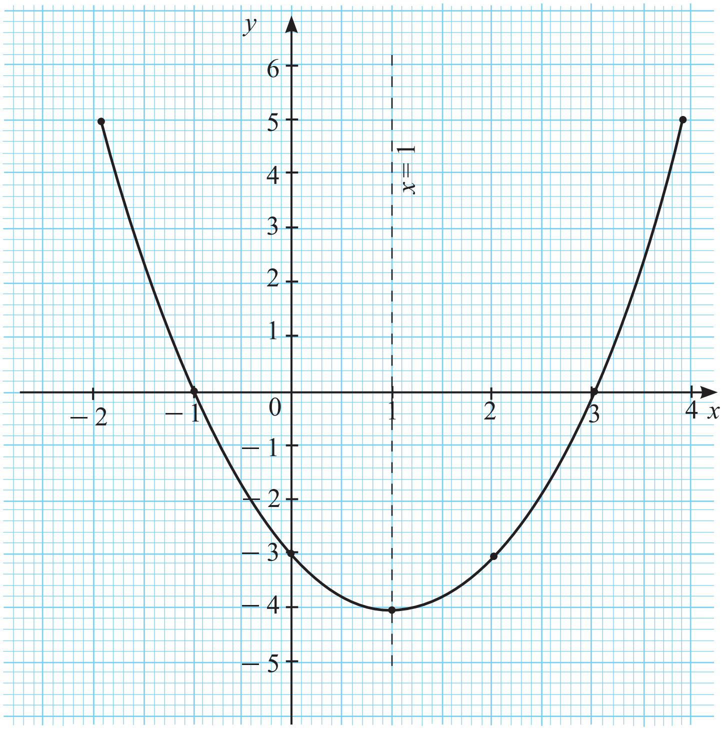
Let us first consider how we can use the knowledge we have gained on the characteristics of graphs of quadratic functions of the form $y = ax^2 + b$ to study the characteristics of graphs of quadratic functions of the form $y = ax^2 + bx + c$.

### Drawing graphs of functions of the form $y = ax^2 + bx + c$ for $a > 0$ and identifying their characteristics

To identify some basic characteristics, let us first draw the graph of the function $y = x^2 - 2x - 3$. To do this, let us prepare a table as follows to obtain the values of $y$ corresponding to the values of $x$ for $-2 \leq x \leq 4$.

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $x^2$ | 4 | 1 | 0 | 1 | 4 | 9 | 16 |
| $-2x$ | 4 | 2 | 0 | $-2$ | $-4$ | $-6$ | $-8$ |
| $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ |
| $y$ | 5 | 0 | $-3$ | $-4$ | $-3$ | 0 | 5 |
| $(x, y)$ | $(-2, 5)$ | $(-1, 0)$ | $(0, -3)$ | $(1, -4)$ | $(2, -3)$ | $(3, 0)$ | $(4, 5)$ |

Before drawing the graph, it is important to consider the range of values that $x$ and $y$ take and prepare the coordinate plane accordingly. It is easy to draw the graph of $y = x^2 - 2x - 3$ by taking 10 small divisions along the $x$-axis to be one unit and 10 small divisions along the $y$-axis to be two units as scale.




The graph of a function of the form $y = ax^2 + bx + c$ is called a parabola.

We can identify the following characteristics by considering the graph.

- The graph is symmetric about the line $x = 1$. Therefore, the equation of the axis of symmetry of the graph is $x = 1$.

As the value of $x$ increases gradually from $-2$, the corresponding $y$ value decreases gradually until it attains its minimum value $-4$ and then starts increasing again.

Let us describe the behaviour of $y$ further for the given range of values of $x$.

- As the value of $x$ increases from $-2$ to $-1$, the value of $y$, that is, the value of the function decreases positively from 5 to 0. What is meant by "decreases positively" is that the function decreases while remaining positive.
- The function takes the value 0 when $x = -1$.
- As the value of $x$ increases from $-1$ to 1, the corresponding $y$ value decreases negatively from 0 to $-4$.
- As the value of $x$ increases from 1 to 3, the corresponding $y$ value increases negatively from $-4$ to 0 .
- The function takes the value 0 when $x = 3$.
- As the value of $x$ increases from 3, the value of $y$ increases positively.

By considering the above characteristics, the range of values of $x$ for which the function is negative can be expressed in terms of inequalities as $-1 < x < 3$.

- The value of $y$ is positive when the value of $x$ is less than $-1$ or greater than 3. That is, the range of values of $x$ for which the function is positive is $x < -1$ and $x > 3$.

Let us also pay attention to the following facts.

- It is very important to understand the relationship between the graph that has been drawn and the function $y = x^2 - 2x - 3$.

This can be described as follows.

(1) If any point $(a, b)$ lies on the graph, then the equation $y = x^2 - 2x - 3$ is satisfied by $x = a$ and $y = b$. That is the equation $b = a^2 - 2a - 3$ is true.

(2) Conversely, if $x = a$ and $y = b$ satisfies the equation $y = x^2 - 2x - 3$, then the point $(a, b)$ lies on the graph of the function $y = x^2 - 2x - 3$.

It is extremely important to keep the above two facts in mind. Clearly the point $(-1, 0)$ lies on the graph. Therefore, $x = -1$ and $y = 0$ should satisfy the equation $y = x^2 - 2x - 3$. That is, $0 = (-1)^2 - 2(-1) - 3$. This can be verified by simplifying the right hand side. Another way of stating this is that $x = -1$ is a root of the equation $x^2 - 2x - 3 = 0$. By the same argument it can be stated that $x = 3$ is also a root of this equation. That is, the roots of the equation $x^2 - 2x - 3 = 0$ are the $x$ coordinates of the points at which the graph of $y = x^2 - 2x - 3$ intersects the $x$- axis. This can be written more generally as follows.
The $x$ coordinates of the points at which the graph of $y = ax^2 + bx + c$ intersects the $x$ - axis are the roots of the quadratic equation $ax^2 + bx + c = 0$.

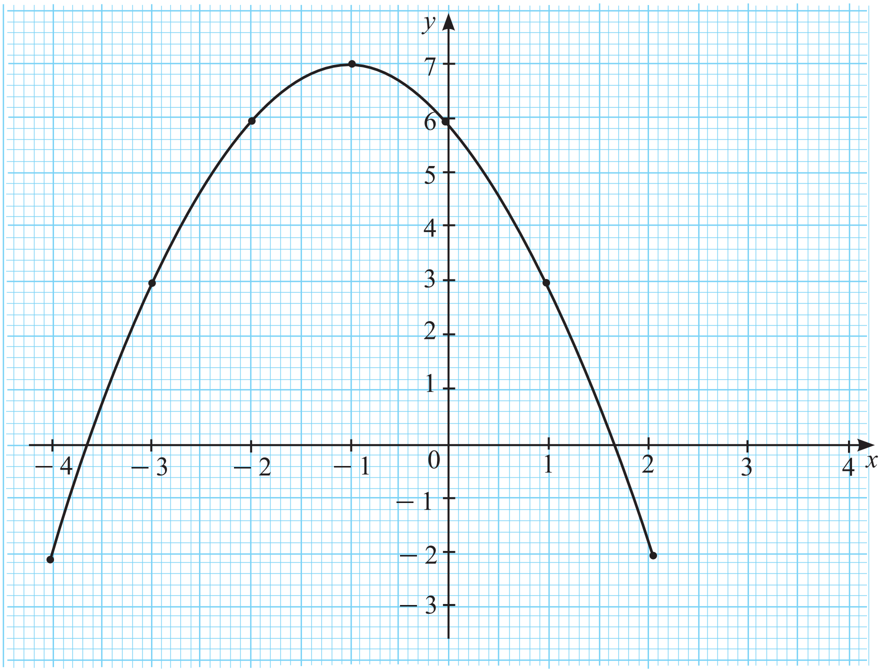
- The minimum value of the function is obtained at the turning point of the graph. The minimum value is $-4$. The coordinates of the minimum point are $(1, -4)$.

## Drawing the graphs of functions of the form $y = ax^2 + bx + c$ for $a < 0$ and identifying their characteristics

Let us prepare a table as follows for $-4 \leq x \leq 2$ to draw the graph of the function $y = -x^2 - 2x + 6$.

| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|---|---|
| $-x^2$ | $-16$ | $-9$ | $-4$ | $-1$ | $0$ | $-1$ | $-4$ |
| $-2x$ | $8$ | $6$ | $4$ | $2$ | $0$ | $-2$ | $-4$ |
| $+6$ | $+6$ | $+6$ | $+6$ | $+6$ | $+6$ | $+6$ | $+6$ |
| $y$ | $-2$ | $3$ | $6$ | $7$ | $6$ | $3$ | $-2$ |
| $(x, y)$ | $(-4, -2)$ | $(-3, 3)$ | $(-2, 6)$ | $(-1, 7)$ | $(0, 6)$ | $(1, 3)$ | $(2, -2)$ |

By considering the range of values of $x$ and $y$, and selecting 10 small divisions along the $x$-axis to be one unit and 10 small division along the $y$ axis to be two units as scale, the graph of the given function can be drawn as follows.




We can identify the following characteristics by considering the graph.

- The graph which has a maximum value 7 is symmetric about the line $x = -1$.
- Therefore, the equation of the axis of symmetry of the graph is $x = -1$.
- The coordinates of the turning point are $(-1, 7)$.
- As the value of $x$ increases from $-4$ to $-3.6$, the value of $y$ increases negatively.
- The function takes the value 0 when $x = -3.6$.
- As the value of $x$ increases from $-3.6$ to $-1$, the corresponding $y$ value increases positively from 0 to 7.
- The function attains its maximum value of $+7$ when the value of $x$ is $-1$.
- As the value of $x$ increases from $-1$ to $+1.6$, the value of the function decreases positively.
- The function takes the value 0 when $x = +1.6$.
- As the value of $x$ increases from $+1.6$, the value of $y$ decreases negatively.
- When the value of $x$ lies between $-3.6$ and $+1.6$, the value of the function is positive. That is, the range of values of $x$ for which the function is positive is $-3.6 < x < +1.6$.
- When the value of $x$ is less than $-3.6$ or greater than $+1.6$, the function is negative.
- That is, the range of values of $x$ for which the function is negative is $x < -3.6$ and $x > +1.6$.

- The graph intersects the line $y = 0$ ($x$-axis) when $x = -3.6$ and $x = +1.6$. Therefore, the values of $x$ which satisfy the equation $-x^2 - 2x + 6 = 0$, that is, the roots of this equation are $x = -3.6$ and $x = +1.6$.
- The maximum and minimum values that the function takes when $0 \leq x \leq 2$, are 6 and $-2$ respectively.

## Exercise 12.2

**1.** Draw the graph of the function in the given range by selecting a suitable scale.

$y = x^2 + 2x - 7 \quad (-4 \leq x \leq 2)$

Using your graph, write down,

(a) the minimum value of the function
(b) the coordinates of the turning point,
(c) the equation of the axis of symmetry after drawing it,
(d) the values of $x$ for which $y = 0$,
(e) the range of values of $x$ for which the function is negative,
(f) the range of values of $x$ for which the function is positive,
(g) the range of values of $x$ for which the function is decreasing positively,
(h) the range of values of $x$ for which the function is increasing negatively.

**2.** An incomplete table of values prepared to sketch the graph of $y = x^2 - 4x + 2$ is given below.

| $x$ | $-1$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| $y$ | ___ | 2 | $-1$ | ___ | $-1$ | 2 | 7 |

(i) Complete the above table and by taking 10 small divisions along the $x$-axis and $y$-axis to be one unit as scale, draw the graph of the given function.

(ii) By considering the graph, write down,
(a) the coordinates of the turning point,
(b) the minimum value of the function,
(c) the values of $x$ for which the function is zero,
(d) the range of values of $x$ for which $y \leq -1$,
(e) the roots of the equation $x^2 - 4x + 2 = 0$,

**3.** For the following function, draw the graph in the given range by selecting a suitable scale.

$y = -x^2 - 2x + 3 \ (-4 \leq x \leq 2)$

Using your graph write down,
(a) the maximum value of the function,
(b) the coordinates of the turning point,

(c) the equation of the axis of symmetry after drawing it,
(d) the values of $x$ for which $y = 0$,
(e) the range of values of $x$ for which the function is positive,
(f) the range of values of $x$ for which the function is negative,
(g) the range of values of $x$ for which the function is increasing positively,
(h) the range of values of $x$ for which the function is decreasing negatively.

**4.** An incomplete table of values of $x$ and $y$ prepared to draw the graph of the function $y = -2x^2 + 3x + 2$ is given below.

| $x$ | $-2$ | $-1$ | 0 | $\frac{3}{4}$ | 1 | 2 | 3 | 3.5 |
|---|---|---|---|---|---|---|---|---|
| $y$ | $-12$ | $-3$ | 2 | ____ | 3 | ____ | $-7$ | $-12$ |

(i) Complete the above table and by taking 10 small divisions along the $x$-axis and $y$-axis to be one unit as scale, draw the graph of the given function.

(ii) By considering the graph, write down
(a) the coordinates of the turning point,
(b) the equation of the axis of symmetry,
(c) the roots of the equation $-2x^2 + 3x + 2 = 0$,
(d) the range of values of $x$ for which the function is increasing positively,
(e) the value of $x$ for which the value of the function is 4,
(f) the values of $x$ for which $y$ is $-4$.

## 12.3 The graph of a function of the form $y = \pm (x + b)^2 + c$.

$y = \pm (x + b)^2 + c$ is also an equation of a quadratic function. Here the quadratic function has been expressed in the special form $y = \pm (x + b)^2 + c$. When it is written in this form, some of the characteristics of its graph can be inferred without drawing the graph. Several such characteristic are given in the following table.

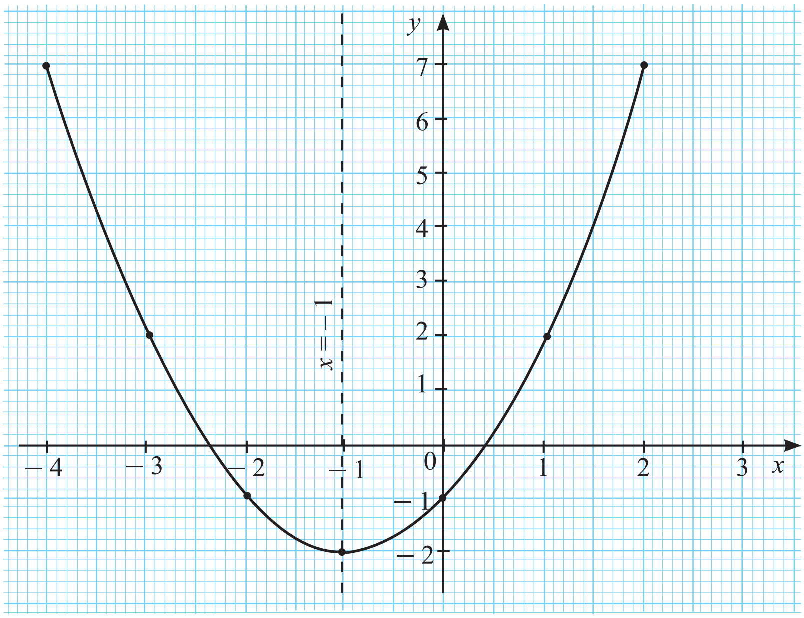
| Equation of the function | Nature of the turning point | Maximum/ minimum value of the function | Coordinates of the maximum/ minimum point of the graph | Equation of the axis of symmetry of the graph | The point at which the function intersects the $y$-axis |
|---|---|---|---|---|---|
| $y = (x + b)^2 + c$ | Minimum | $c$ | $(-b, c)$ | $x = -b$ | $(0, b^2 + c)$ |
| $y = -(x + b)^2 + c$ | Maximum | $c$ | $(-b, c)$ | $x = -b$ | $(0, -b^2 + c)$ |

To verify the characteristics in the above table, let us consider the following example.

Let us consider the function $y = (x + 1)^2 - 2$. This is of the form $y = (x + b)^2 + c$ where $b = 1$ and $c = -2$. To draw the graph of this function for the values of $x$ from $-4$ to $+2$, let us calculate the necessary values of $y$ as shown in the following table.

| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|---|---|
| $(x+1)^2$ | 9 | 4 | 1 | 0 | 1 | 4 | 9 |
| $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ |
| $y$ | 7 | 2 | $-1$ | $-2$ | $-1$ | 2 | 7 |
| $(x, y)$ | $(-4, 7)$ | $(-3, 2)$ | $(-2, -1)$ | $(-1, -2)$ | $(0, -1)$ | $(1, 2)$ | $(2, 7)$ |

By taking 10 small divisions along the $x$-axis to be one unit and 10 small divisions along the $y$-axis to be two units as scale, the graph of the given function can be drawn as follows.



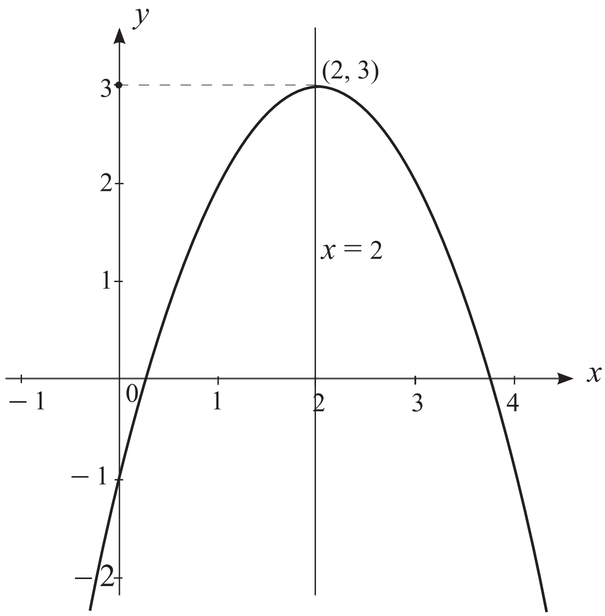

**Note:**
This graph has a minimum point. The minimum value of the function is $-2$ $(= c)$. The coordinates of the minimum point are $(-1, -2)$, that is, $(-b, c)$. The axis of symmetry is $x = -1$ (That is, $x = -b$).

When a quadratic function has been given in the form $x = \pm (x + b)^2 - c$, the graph can be sketched using the characteristics in the above table. The example given below explains how this is done.

**Example 1**

Sketch the graph of $y = -\ (x-2)^2 + 3$.

Since the coefficient of $(x-2)^2$ is negative, the turning point of the graph is a maximum. The coordinates of this maximum point is (2, 3). The axis of symmetry of the graph is $x = 2$. Further, to find the point at which the graph intersects the $y$-axis, let us substitute $x = 0$ in $y = -\ (x-2)^2 + 3$. We then obtain $y = -(0-2)^2 + 3 = -1$. Accordingly, we can sketch the graph as follows.




**Example 2**

Write down the following for the function $y = x^2 + 3x - 4$.

(i) The nature of the graph
(ii) The equation of the axis of symmetry
(iii) The maximum/minimum value
(iv) The coordinates of the turning point of the graph

The function has been given in the form $y = ax^2 + bx + c$.
Let us first write it in the form $y = (x + k)^2 + h$.

$$y = x^2 + 3x - 4$$

$$y = (x + \frac{3}{2})^2 - 4 - \frac{9}{4}$$

$y = (x + \frac{3}{2})^2 - \frac{25}{4}$

(i) A parabola with a minimum.

(ii) $x = -\frac{3}{2}$. That is, $x = -1\frac{1}{2}$.

(iii) Minimum value is $-\frac{25}{4}$.

(iv) $(-\frac{3}{2}, -\frac{25}{4})$

**Exercise 12.3**

**1.** Sketch the graph of each of the following functions in the given interval by selecting a suitable scale.

(i) $y = (x - 2)^2 - 3 \quad (-1 \leq x \leq 5)$ (ii) $y = (x + 3)^2 - 4 \quad (-6 \leq x \leq 0)$

By considering each graph, write down

**a.** the minimum value of the function,
**b.** the coordinates of the minimum point of the graph,
**c.** the equation of the axis of symmetry after drawing it,
**d.** the range of values of $x$ for which the function is positive,
**e.** the values of $x$ for which $y = 0$,
**f.** the range of values of $x$ for which the function is negative.

**2.** Sketch the graph of each of the following functions in the given interval by selecting a suitable scale.

(i) $y = -(x + 2)^2 + 2 \quad (-5 \leq x \leq 1)$ (ii) $y = -(x - 1)^2 + 3 \quad (-2 \leq x \leq 4)$

By considering each graph, write down

**a.** the maximum value of the function,
**b.** the coordinates of the maximum point of the graph,
**c.** the equation of the axis of symmetry after drawing it,
**d.** the range of values of $x$ for which the function is positive,
**e.** the range of values of $x$ for which the function is negative,
**f.** the values of $x$ for which $y = 0$,
**g.** the range of values of $x$ for which the function is increasing positively,
**h.** the range of values of $x$ for which the function is decreasing negatively.

**3.** Draw a rough sketch of the graph of each of the functions.

(i) $y = (x - 2)^2 - 3$

(ii) $y = 2 - (x + 5)^2$

(iii) $y = x^2 + 6x - 1$

**4.** Without sketching the graph, write down the following for each of the given functions.

**a.** Nature of the graph
**b.** Equation of the axis of symmetry
**c.** Maximum/minimum value
**d.** Coordinates of the turning point

(i) $y = (x + 2)^2 - 3$ (ii) $y = -(x - 2)^2 + 4$ (iii) $y = -(x - \frac{3}{2})^2 + 1$

(iv) $y = 1\frac{1}{2} - (x - \frac{1}{2})^2$ (v) $y = 3\frac{1}{3} + (x + 2\frac{1}{2})^2$ (vi) $y = x^2 + 6x + 5$

## 12.4 Graphs of functions of the form $y = \pm (x \pm a)(x \pm b)$

$y = \pm (x + a)(x + b)$ is also an equation of a quadratic function. The function has been written in the special form $y = \ x \pm (x + a)(x + b)$. As in the above case, when the function is given in this form, certain characteristics of the graph of the function can be inferred without sketching the graph. The following table provides several such characteristics.

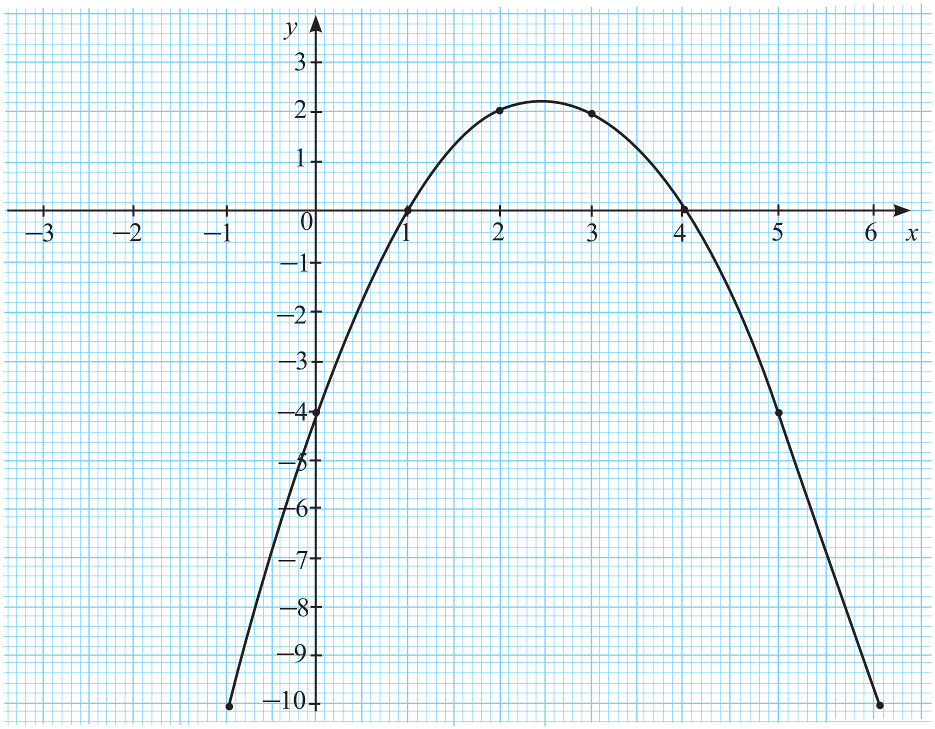
| Equation of the function | Nature of the turning point | The coordinates of the maximum/ minimum point | Equation of the axis of symmetry of the graph | The points at which the function intersects the $x$-axis | The point at which the function intersects the $y$-axis |
|---|---|---|---|---|---|
| $y = (x + a)(x + b)$ | Minimum | $\left(-\frac{(a + b)}{2}, -\frac{(a - b)^2}{4}\right)$ | $x = -\frac{(a + b)}{2}$ | $(-a, 0)$ and $(-b, 0)$ | $(0, ab)$ |
| $y = -(x + a)(x + b)$ | Maximum | $\left(-\frac{(a + b)}{2}, \frac{(a - b)^2}{4}\right)$ | $x = -\frac{(a + b)}{2}$ | $(-a, 0)$ and $(-b, 0)$ | $(0, -ab)$ |

To verify the characteristics in the above table let us consider the following example.

Let us consider $y = -(x - 1)(x - 4)$. This is of the form $y = -(x + a)(x + b)$ where $a = -1$ and $b = -4$. To draw the graph of this function, let us prepare a table of $(x, y)$ values as follows.

| $x$ | – 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|
| $-(x-1)(x-4)$ | – 10 | – 4 | 0 | 2 | 2 | 0 | – 4 | – 10 |
| $(x, y)$ | (– 1, – 10) | (0, – 4) | (1, 0) | (2, 2) | (3, 2) | (4, 0) | (5, – 4) | (6, – 10) |

By taking 10 small divisions along the $x$-axis to be one unit and 10 small divisions along the $y$-axis to be two units, as scale, the graph of the given function can be drawn as follows.




Verify as in the example of section 12.3 that this graph has the characteristics given in the table.

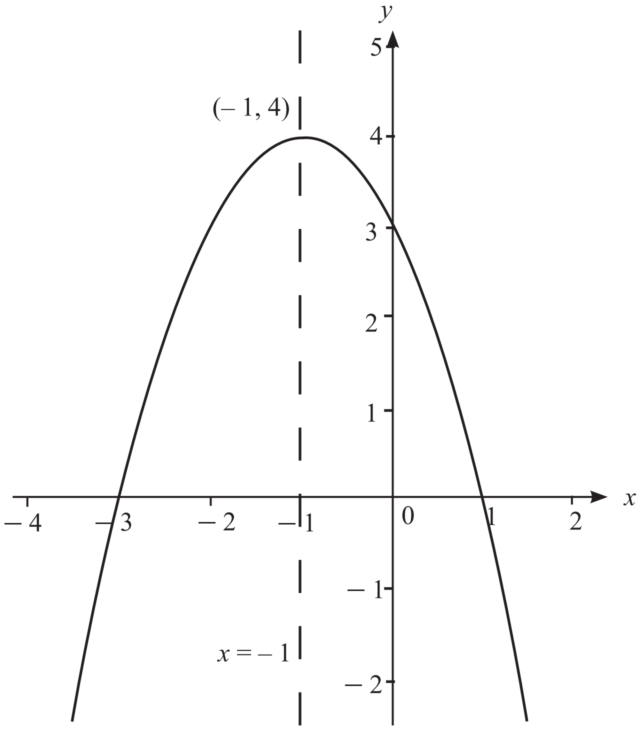
When a quadratic function is given in the form $y = \pm (x + a)(x + b)$, its graph can be sketched by considering the characteristics given in the table.

The following example describes how this is done.

**Example 1**

Sketch the graph of $y = -(x + 3)(x - 1)$.

This is of the form $y = -(x + a)(x + b)$, where $a = 3$ and $b = -1$ Since the coefficient of $x^2$ is negative, the turning point is a maximum. The points at which the graph intersects the $x$-axis are (– 3, 0) and (1, 0). The coordinates of the maximum point are $\left(-\frac{a+b}{2}, \frac{+(a-b)^2}{4}\right) = (-1,\ 4)$. Accordingly, a graph of the following form can be drawn.




**Example 2**

Without drawing the graph of the function $y = x^2 + 5x - 14$, write down the following.

(i) The nature of the graph
(ii) The equation of the axis of symmetry
(iii) The maximum/minimum value
(iv) The coordinates of the turning point
(v) The coordinates of the points at which the graph intersects the $x$-axis

Let us organize this function in the form $y = (x + a)(x + b)$.
The given function can be written as $y = (x - 2)(x + 7)$.

(i) The graph is a parabola with a minimum.
(ii) Since $a = -2$ and $b = 7$, the axis of symmetry is

$$x = \frac{-(a+b)}{2} = \frac{-(-2+7)}{2};\ \text{that is, } x = -\frac{5}{2}.$$

(iii) Since the minimum value is obtained from $\frac{-(a-b)^2}{4}$,

the minimum value is $\frac{-(-2-7)^2}{4} = -\frac{81}{4}$.

(iv) The coordinates of the minimum point are $(-\frac{5}{2}, -\frac{81}{4})$.

(v) The coordinates of the intersection point of the graph and the $x$-axis are given by $(-a, 0)$ and $(-b, 0)$; that is, $(2, 0)$ and $(-7, 0)$.

**Exercise 12.4**

**1.** Sketch the graph of each of the given functions for the given range of values of $x$ by selecting a suitable scale.

(a) $y = (x+1)(x+6)$ $(-7 \leq x \leq 0)$
(b) $y = (x-2)(x-5)$ $(0 \leq x \leq 7)$
(c) $y = -(x+1)(x+3)$ $(-5 \leq x \leq 1)$
(d) $y = -(x-5)(x-3)$ $(+1 \leq x \leq 7)$

For each of the graphs, write down the following.

(i) The values of $x$ for which $y$ is zero.
(ii) The equation of the axis of symmetry after drawing it.
(iii) The minimum/maximum value of the function.
(iv) The coordinates of the minimum/maximum point.
(v) The range of values of $x$ for which the function is positive.
(vi) The range of values of $x$ for which the function is negative.
(vii) The behaviour of $y$ as $x$ increases within the given range.

**2.** Draw a rough sketch of the graph of each of the functions.

(i) $y = (x-3)(x+5)$
(ii) $y = (x-1)(x-2)$
(iii) $y = -(x+3)(x-6)$

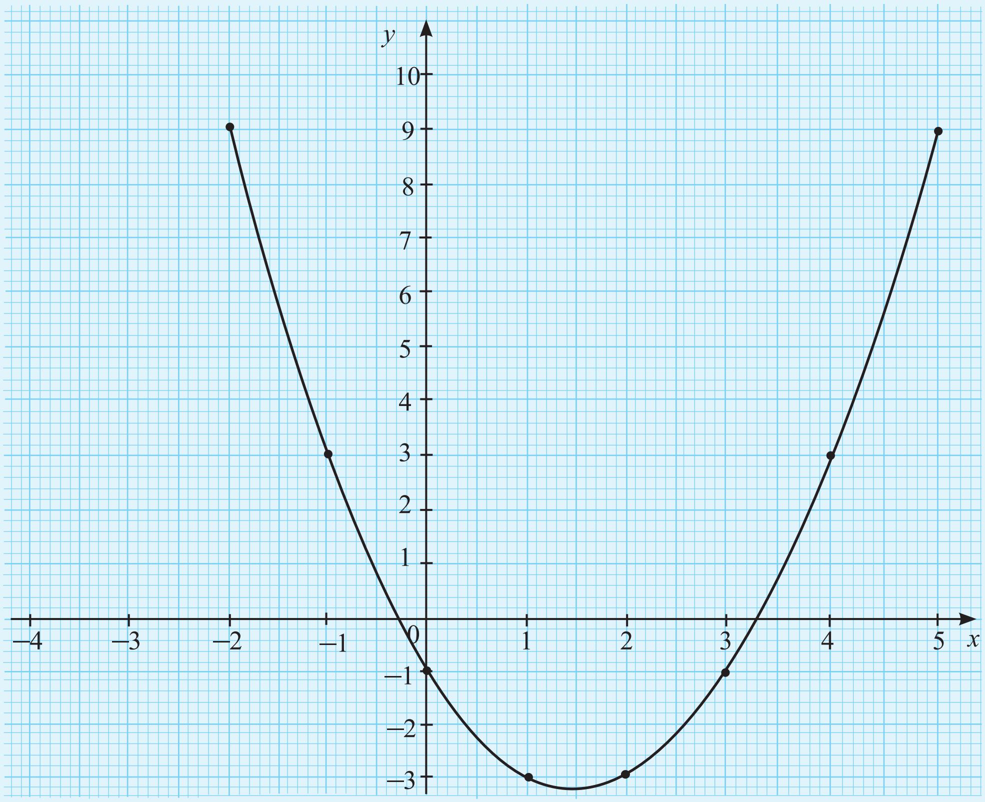
**3.** Without sketching the graph, write down the following for each of the given functions.

**a.** Nature of the graph **b.** Equation of the axis of symmetry
**c.** Maximum/minimum value **d.** Coordinates of the turning point.

(i) $y = (x-2)(x+3)$ (ii) $y = (x+1)(x-4)$ (iii) $y = (x-4)(x-1)$
(iv) $y = -(x-\frac{1}{2})(x+3)$ (v) $y = x^2 - 1\frac{1}{2}x - 2\frac{1}{2}$ (vi) $y = x^2 - 4x + 7$
(vii) $y = -x^2 - 6x - 5$ (viii) $y = -x^2 + 12x + 35$ (ix) $y = x^2 - x + 4$

## Miscellaneous Exercise

**1. (*a*)** A sketch of the graph of a quadratic function for values of $x$ such that $-2 \leq x \leq 5$ is given below.




Answer the following by considering this graph.

(i) Find the value of $y$ when $x = 3$.

(ii) Draw the axis of symmetry and write down its equation.

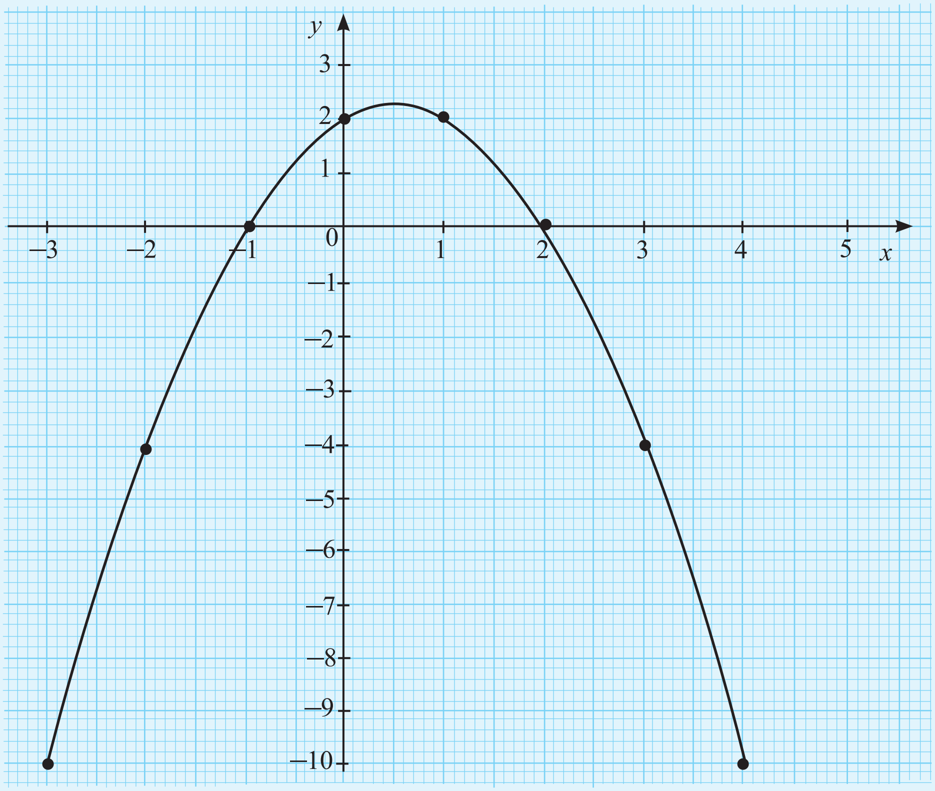
(iii) Write down the interval of values of $x$ for which the function is negative.

(iv) This function can be expressed in the form $y = (x - a)^2 + b$. Determine the values of $a$ and $b$.

(v) For the function obtained in (iv), find the values of $x$ for which $y = 0$.

(vi) Write down the function which has the same axis of symmetry as in (ii), a maximum value of 5 and coefficient of $x^2$ equal to 1.

**(*b*)** A sketch of the graph of a quadratic function for values of $x$ such that $-3 \leq x \leq 4$ is given below.




(i) Write down the values of $x$ for which $y = 0$.

(ii) The relevant function can be expressed in the form $y = -(x - a)(x - b)$. By considering the answer to (i) above, determine the values of $a$ and $b$.

(iii) Express the equation of the function obtained in (ii) above by substituting the values of $a$ and $b$, in the form $y = -(x - p)^2 + q$ and obtain the coordinates of the maximum point. Verify your answer by considering the graph.

(iv) Write down the interval of values of $x$ for which $y \leq -4$.

(v) Write down the interval of values of $x$ for which the function is increasing positively.

**2.** $(x + 2)$ and $(3 - x)$ represent two numbers. $y = (x + 2)(3 - x)$ denotes the product of these two numbers.

(i) Complete the following table.

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|---|
| $y$ | $-6$ | ___ | ___ | $6$ | ___ | $4$ | ___ | $-6$ |

(ii) Sketch the graph of the function $y$ by selecting a suitable scale.

Answer the following questions by considering the graph.

(iii) Find the maximum value of the product.

(iv) Find the value of $x$ for which the product is maximum.
(v) Write down the values of $x$ for which the product is zero.
(vi) Write down the interval of values of $x$ for which $y > 3$.
(vi) For which interval of values of $x$ does the product increase gradually?
(viii) For which interval of values of $x$ is the product positive?
(ix) Write down the maximum and minimum values of the product for $-1 \leq x \leq 3$.
(ix) Write down the maximum and minimum values of the product for $5 \leq x \leq 8$.

**3.** An incomplete table of several values of $x$ and corresponding values of the function $y = (x-2)^2 - 2$ are given below.

| $x$ | $-1$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| $y$ | 7 | 2 | $-1$ | $-2$ | ___ | 2 | 7 |

(i) Find the value of $y$ when $x = -2$.
(ii) Sketch the graph of the given function by selecting a suitable scale.
(iii) Write down the coordinates of the turning point.
(iv) Write down the interval of values of $x$ for which $y < 0$.
(v) By using the graph and algebraically, find the roots of the equation $x^2 - 4x + 2 = 0$ and hence obtain an approximate value for $\sqrt{2}$.
(vi) Write down the values of $x$ for which the value of the function is 3.

**4.** An incomplete table of $x$ and $y$ values suitable to sketch the graph of $y = -(x+1)(x-3)$ is given below.

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $y$ | ___ | 0 | 3 | 4 | 3 | ___ | $-5$ |

(i) Find the value of $y$ when $x = -2$ and $x = 3$.
(ii) Sketch the graph of the given function by selecting a suitable scale.
(iii) Write down the coordinates of the maximum point.
(iv) Obtain the values of $x$ for which $y = 0$, and thereby verify that the maximum value is correct.
(v) Write down the interval of values of $x$ for which $y \geq -1$.
(vi) Write down the roots of the equation $-x^2 + 2x + 3 = 0$.
(vii) Describe the behavior of the function for $1 \leq x \leq 4$.

**5.** An incomplete table of $x$ and $y$ values suitable to sketch the graph of $y = 5 - x - x^2$ is given below.

| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|
| $y$ | ___ | $-1$ | 3 | 5 | 5 | ___ | $-1$ | $-7$ |

(i) Find the value of $y$ when $x = -4$ and $x = 1$.

(ii) Sketch the graph of the given function by selecting a suitable scale.

(iii) Write down the coordinates of the maximum point.

(iv) Write down the interval of values of $x$ in which the function increases from $-5$ to $+3$.

(v) Write down the interval of values of $x$ for which the function is negative.

(vi) Write down the roots of the equation $-x^2 - x + 5 = 0$ by considering the graph.

(vii) Deduce the coordinates of the maximum point of the function given by $y - 3 = 5 - x - x^2$.

# 13 Equations

**By studying this lesson you will be able to:**

- construct simultaneous equations with rational coefficients,
- solve simultaneous equations with rational coefficients,
- solve quadratic equations by means of factoring, completing the square and using the formula.

## Solving Simultaneous Equations

To review what you have learnt so far on solving simultaneous equations, do the following problems.

**Review Exercise**

**1.** Solve the following pairs of simultaneous equations.

**a.** $6x + 2y = 1$
$4x - y = 3$

**b.** $a + 2b = 3$
$2a + 3b = 4$

**c.** $m - 4n = 6$
$3m + 2n = 4$

**d.** $9p - 2q = 13$
$7p - 3q = 0$

**e.** $2x + 3y = 12$
$3x - 4y = 1$

**f.** $3a + 12 = 2b$
$13 + 2a = 3b$

**2.** Sarath has twenty, two rupee and five rupee coins, which total Rs 55. Let $x$ be the number of two rupee coins and $y$ be the number of five rupee coins Sarath has.
(i) Express the given information in two equations.
(ii) How many of each type of coin does he have?

**3.** Malini and Nalini have a certain amount of money. When you add Rs 30 to the sum of the amounts Malini and Nalini have, the total amounts to Rs 175. Nalini has Rs 95 less than twice the amount Malini has. Let Rs $x$ be the amount of money Malini has and Rs $y$ be the amount of money Nalini has,
(i) Express the given information in two equations.
(ii) How much does each person actually have?

**4.** "It costs Rs 65 to buy two books and a pen. You can buy one such book at the cost of two such pens." Construct two simultaneous equations to represent this information and solve them to find the price of a book and a pen separately.

## 13.1 Simultaneous equations with fractional coefficients

We have learnt before how to solve a pair of simultaneous equations when the unknowns have integers as coefficients. Now we will explore through an example, how to solve a pair of simultaneous equations when the unknowns have fractions as coefficients.

**Example 1**

Kamal and Nimal have a certain amount of money. When you add $\frac{1}{2}$ of the amount Kamal has to $\frac{1}{3}$ of the amount Nimal has to get Rs 20. If $\frac{1}{4}$ th of what Kamal has is equal to $\frac{1}{6}$ th of what Nimal has, find the amounts Nimal and Kamal have separately.

Let us see how we can construct a pair of simultaneous equations to solve this problem.

Let us take the amount Kamal has as Rs $x$ and the amount Nimal has as Rs $y$. Then, when you add $\frac{1}{2}$ of what Kamal has, that is, Rs $\frac{1}{2}x$ to $\frac{1}{3}$ of what Nimal has, that is, $\frac{1}{3}y$, you get $\frac{1}{2}x + \frac{1}{3}y$. Since this is equal to Rs 20 we get

$$\frac{1}{2}x + \frac{1}{3}y = 20. \text{ ——— ① as one equation.}$$

Similarly , since $\frac{1}{4}$ th of what Kamal has is equal to $\frac{1}{6}$ th of what Nimal has, we get

$\frac{1}{4}x = \frac{1}{6}y$ as the second equation, which can be written as

$$\frac{1}{4}x - \frac{1}{6}y = 0 \text{ ——— ②}$$

When solving simultaneous equations involving fractions, it often helps to first clear the fractions and work with only integers. To clear the fractions we have to multiply each side of the equation by the least common multiple of the denominators.

Therefore, equation ① is multiplied by 6 which is the the least common multiple of the denominators 2, and 3 and equation ② is multiplied by 12 which is the least common multiple of the denominators 6 and 4.

$$① \times 6;\ 6 \times \frac{1}{2}x + 6 \times \frac{1}{3}y = 6 \times 20$$

$$\therefore\ 3x + 2y = 120 \text{ ——— } ③$$

$$② \times 12;\ 12 \times \frac{1}{4}x - 12 \times \frac{1}{6}y = 12 \times 0$$

$$3x - 2y = 0 \text{ ——— } ④$$

Now instead of ① and ② we can solve the two equivalent equations ③ and ④ involving only integers.

$$③ + ④ \quad (3x + 2y) + (3x - 2y) = 120 + 0$$

$$3x + 2y + 3x - 2y = 120$$

$$\frac{6x}{6} = \frac{120}{6}$$

$$x = 20$$

By substituting $x = 20$ in ④

$$3 \times 20 - 2y = 0$$

$$2y = 60$$

$$y = 30$$

$\therefore$ The amount Kamal has = Rs 20

The amount Nimal has = Rs 30

Note: In this problem, once we converted the fractions into integers, we solved for the unknown $x$ by adding the equations. Alternatively we can make one unknown the subject of one equation and substitute in the other equation to obtain the solution. We will discuss one such example now.

**Example 2**

Solve.

$$\frac{1}{6}a - \frac{1}{5}b = -2 \text{ ——— } ①$$

$$\frac{1}{3}a + \frac{1}{4}b = 9 \text{ ——— } ②$$

The following steps will demonstrate how to solve simultaneous equations by substitution.

Let us make $a$ the subject of equation ①.

$$\frac{1}{6}a - \frac{1}{5}b = -2$$

$$\frac{1}{6}a = -2 + \frac{1}{5}b$$

$$a = -12 + \frac{6}{5}b \quad \text{(multiplying both sides by 6)} \text{———} ③$$

We will substitute the value of $a$ in equation ②.

$$\frac{1}{3}a + \frac{1}{4}b = 9$$

$$\frac{1}{3}(-12 + \frac{6}{5}b) + \frac{1}{4}b = 9$$

$$-4 + \frac{2}{5}b + \frac{1}{4}b = 9$$

We will simplify the fractions by taking 20 as the common denominator which is the least common multiple of 4 and 5.

$$\frac{8}{20}b + \frac{5}{20}b = 9 + 4$$

$$\frac{13}{20}b = 13$$

$$b = \frac{13 \times 20}{13}$$

$$b = 20$$

Substituting $b = 20$ in equation ③ (here you can substitute the value of $b$ in either equation to find $a$), we get

$$a = -12 + \frac{6}{5}b$$

$$a = -12 + \frac{6}{5} \times 20$$

$$a = -12 + 24$$

$$a = 12$$

Therefore the solution to the problem is $a = 12$ and $b = 20$.

We can verify that the solution is correct by substituting $a = 12$ and $b = 20$ in the original equations.

Let us substitute $a = 12$ and $b = 20$ in the left side of equation ①.

$$\frac{1}{6}a - \frac{1}{5}b = -2$$

$$\text{Left hand side } = \frac{1}{6}a - \frac{1}{5}b$$

$$= \frac{1}{6} \times 12 - \frac{1}{5} \times 20$$

$$= 2 - 4$$

$$= -2$$

$\therefore$ LHS = RHS

The left hand side equals the right hand side of the equation after the substitution.

That means $a = 12$ and $b = 20$ satisfy the equation $\frac{1}{6}a - \frac{1}{5}b = -2$

Similarly,
let us substitute $a = 12$ and $b = 20$ in the left side of equation ②.

$$\frac{1}{3}a + \frac{1}{4}b = 9$$

$$\text{Left hand side } = \frac{1}{3}a + \frac{1}{4}b$$

$$= \frac{1}{3} \times 12 + \frac{1}{4} \times 20$$

$$= 4 + 5$$

$$= 9$$

$\therefore$ LHS = RHS

The left hand side equals the right hand side of the equation after the substitution.

That means $a = 12$ and $b = 20$ satisfy the equation $\frac{1}{3}a + \frac{1}{4}b = 9$.

Therefore we can conclude that we have the correct answer.

**Example 3**

Solve :

$$\frac{1}{2}m + \frac{2}{3}n = 1$$

$$\frac{5}{6}m + \frac{1}{3}n = 4$$

Let us take:

$\frac{1}{2}m + \frac{2}{3}n = 1$ ——— ①

$\frac{5}{6}m + \frac{1}{3}n = 4$ ——— ②

As in example 1 we can either convert the fractions into integers or we can equate the fractional coefficients of a variable to solve the problem.

Let us equate the coefficients of the unknown $n$. We can do this by multiplying equation ② by 2

② × 2 $\quad \frac{10}{6}m + \frac{2}{3}n = 8$ ——— ③

③ − ① $\quad (\frac{10}{6}m + \frac{2}{3}n) - (\frac{1}{2}m + \frac{2}{3}n) = 8 - 1$

$$\frac{10}{6}m + \frac{2}{3}n - \frac{1}{2}m - \frac{2}{3}n = 7$$

$$\frac{10}{6}m - \frac{3}{6}m = 7$$

$$\frac{7}{6}m = 7$$

$$7m = 7 \times 6$$

$$m = 6$$

Let us substitute $m = 6$ in ①.

$$\frac{1}{2}m + \frac{2}{3}n = 1$$

$$\frac{1}{2} \times 6 + \frac{2}{3}n = 1$$

$$3 + \frac{2}{3}n = 1$$

$$\frac{2}{3}n = 1 - 3$$

$$\frac{2}{3}n = -2$$

$$2n = -6$$

$$n = -3$$

Therefore the answer is $m = 6$ and $n = -3$.

As in the previous problem, we can verify that the solution is correct by substituting the answers in the original equations.

Let us substitute $m = 6$ and $n = -3$ in the original equations.

$$\frac{1}{2}m + \frac{2}{3}n = 1 \text{ ——— ①} \qquad \frac{5}{6}m + \frac{1}{3}n = 4 \text{ ——— ②}$$

$$\begin{aligned}\text{Left hand side } &= \frac{1}{2}m + \frac{2}{3}n \\ &= \frac{1}{2} \times 6 + \frac{2}{3} \times (-3) \\ &= 3 - 2 \\ &= \underline{\underline{1}}\end{aligned}$$

$\therefore$ Left hand side = Right hand side

$$\begin{aligned}\text{Left hand side } &= \frac{5}{6}m + \frac{1}{3}n \\ &= \frac{5}{6} \times 6 + \frac{1}{3} \times (-3) \\ &= 5 - 1 \\ &= \underline{\underline{4}}\end{aligned}$$

$\therefore$ Left hand side = Right hand side

Therefore we have the correct answer, $m = 6$ and $n = -3$.

**Exercise13.1**

**1.** Solve the following pairs of simultaneous equations.

**(a)** $\frac{3}{5}a + \frac{1}{3}b = 3$

$\frac{1}{2}a - \frac{1}{3}b = 8$

**(b)** $\frac{3}{5}x - \frac{1}{2}y = 9$

$\frac{1}{4}x - \frac{1}{2}y = 2$

**(c)** $\frac{1}{3}x + \frac{1}{2}y = 4$

$\frac{1}{2}x - y = 1$

**(d)** $\frac{2}{7}p - \frac{1}{3}q = 5$

$\frac{1}{2}p - 1\frac{2}{3}q = 12$

**(e)** $\frac{m}{4} + \frac{5n}{3} = 36$

$\frac{3m}{8} - \frac{5n}{12} = -2$

**(f)** $\frac{2x}{3} + \frac{3y}{2} = -1$

$4x - 5y = 22$

**2.** At a festival held in a school, the past pupils' association agreed to bear $\frac{1}{2}$ of the amount spent on refreshments and $\frac{1}{3}$ of the amount spent on decorations

Accordingly,the amount given by the past pupils' association was Rs 20 000.Rest of the amount spent on refreshments and decorations was given by the welfare society, which was Rs 30 000.

(i) Taking the amount spent on refreshment as $x$ and the amount spent on decorations as $y$, construct a pair of simultanious equations to indicate the above information.

(ii) Solve the equations and find seperately the amount spent on refreshments and decorations.

## 13.2 Solving quadratic equations by factoring

You have learnt how to find the solutions (or roots) of a quadratic equation of the form $ax^2 + bx + c = 0$

Let us review a few a such examples.

**Example1**

Find the roots of the quadratic equation $x^2 – 5x + 6 = 0$.

$(x – 2)(x – 3) = 0$ (by factoring)

Then either $x – 2 = 0$ or $x – 3 = 0$.

Therefore $x = 2$ or $x = 3$

Therefore the roots of the equation are $x = 2$ and $x = 3$.

**Example 2**

Find the roots of $2x^2 + 3x – 9 = 0$

$$2x^2 + 6x – 3x – 9 = 0$$

$$2x(x + 3) – 3(x + 3) = 0$$

$$(2x – 3)(x + 3) = 0 \text{ (by factoring)}$$

$$2x – 3 = 0 \text{ or } x + 3 = 0$$

$$x = \frac{3}{2} \text{ or } x = – 3$$

$\therefore \quad x = 1\frac{1}{2}$ and $x = – 3$ are the roots of the equation.

Let us look at a more complex example.

**Example 3**

Find the roots of $\dfrac{3}{2x – 1} – \dfrac{2}{3x + 2} = 1$.

Here we cannot see the quadratic equation immediately. But when we clear the fractions, it can be set up as a quadratic equation. For this, we will consider the common denominator of the left hand side.(You can also do this by multiplying the whole equation by the least common multiple of $2x – 1$ and $3x + 2$)

$$\frac{3(3x + 2) – 2(2x – 1)}{(2x – 1)(3x + 2)} = 1 \text{ (writing the left hand side as a single fraction)}$$

$$3(3x+2)-2(2x-1)=(2x-1)(3x+2) \text{ (cross multiplying)}$$
$$9x+6-4x+2=6x^2+4x-3x-2 \text{ (expanding the factors)}$$
$$6x^2-4x-10=0 \text{ (simplifying)}$$
$$3x^2-2x-5=0 \text{ (dividing the whole equation by 2)}$$
$$3x^2-5x+3x-5=0$$
$$x(3x-5)+1(3x-5)=0$$
$$(3x-5)(x+1)=0$$
$$\therefore\ 3x-5=0 \text{ or } x+1=0$$
$$\therefore\ x=\frac{5}{3} \text{ or } x=-1$$
$$\therefore\ x=1\frac{2}{3} \text{ or } x=-1$$
$$\therefore\ x=1\frac{2}{3} \text{ and } x=-1 \text{ are the roots of this equation.}$$

Now let us consider a problem that can be solved by means of a quadratic equation.

**Example 4**

The product of two consecutive integers is 12. Find the two numbers.

Let us see how we can set up a quadratic equation to solve this problem.
Of the two consecutive integers, let us take $x$ to be the smaller integer. Then the other integer is $x+1$.

Therefore the consecutive integers can be written as $x$, $(x+1)$.
Since the product of the two numbers is 12 we get,

$$x\times(x+1)=12.$$
$$\therefore\ x^2+x-12=0$$

After factoring we get

$$(x-3)(x+4)=0.$$

This is, $x-3=0$ or $x+4=0$.
$\therefore$ $x=3$ and $x=-4$ are the solutions of the above equation.

If we take $x=3$ then the consecutive number is $(x+1)=3+1=4$.
If we take $x=-4$ the consecutive number is $(x+1)=-4+1=-3$.

Therefore there are two pairs of consecutive numbers whose product will be 12. They are 3, 4 and – 4, – 3.

We can verify the answer by substituting the two pairs of values in the quadratic equation $x^2+x-12=0$.

If we substitute $x = 3$ on the left hand side of $x^2 + x - 12 = 0$,

$$\begin{aligned} \text{LHS} &= x^2 + x - 12 \\ &= 3^2 + 3 - 12 \\ &= 9 + 3 - 12 \\ &= 12 - 12 \\ &= 0 \end{aligned}$$

$\therefore$ LHS = RHS

If we substitute $x = -4$ on the left hand side,

$$\begin{aligned} \text{LHS} &= x^2 + x - 12 \\ &= (-4)^2 + (-4) - 12 \\ &= 16 - 4 - 12 \\ &= 16 - 16 \\ &= 0 \end{aligned}$$

$\therefore$ LHS = RHS

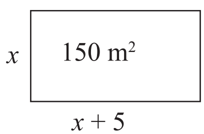
Therefore the solutions of the equation $x^2 + x - 12 = 0$ are indeed 3 and $-4$.

**Example 5**

The area of a rectangular plot of land is 150 square metres. The length of the plot is 5 metres more than the width. Let $x$ be the width of the plot.

(i) Write the length of the plot in terms of $x$.
(ii) Set up a quadratic equation in $x$ to represent the area of the plot $x$.
(iii) Solve the equation to find the length and the width of the plot.

(i) Length $= x + 5$.
(ii) The given data can be shown more clearly by aid of a drawing.




$$\begin{aligned} \text{Area} &= \text{length} \times \text{width} \\ &= (x + 5) \times x \end{aligned}$$

$$x(x + 5) = 150$$

This is the required equation.

(iii) Let us solve the above equation.

$$x(x + 5) = 150$$
$$x^2 + 5x - 150 = 0$$
$$(x - 10)(x + 15) = 0$$

$\therefore\ x - 10 = 0$ or $x + 15 = 0$

$\therefore\ x = +\,10$ or $x = -\,15$

The roots of the quadratic equation are $x = +\,10$ and $x = -\,15$.
But since a length cannot be negative, the only acceptable answer is $x = 10$.

Therefore the width of the rectangular plot = 10 m

The length of the rectangular plot = 15 m

We can verify the answer by substituting $x = 10$ in the quadratic equation $x^2 + 5x - 150 = 0$.

$$\begin{aligned} \text{LHS} &= x\,(x + 5) \\ &= 10\,(10 + 5) \\ &= 10 \times 15 \\ &= 150 \end{aligned}$$

$\therefore$ LHS = RHS

**Exercise 13.2**

**1.** Solve the following quadratic equations.

**(a)** $x\,(x + 5) = 0$ **(b)** $\frac{3}{4}\,x\,(x + 1) = 0$ **(c)** $(x - 4)\,(x + 3) = 0$

**(d)** $x^2 - 2x = 0$ **(e)** $\frac{x^2}{2} = 3x$ **(f)** $x^2 + 7x + 12 = 0$

**(g)** $(x - 2)\,(2x + 3) = x^2 + 2x + 4$ **(h)** $\frac{4}{x} + \frac{3}{x+1} = 3$

**(i)** $\frac{2}{x-1} + \frac{3}{x+1} = 1$ **(j)** $x^2 - 4 = 0$

**2.** Solve each quadratic equation given below by factoring.

(Take $\sqrt{2} = 1.41$, $\sqrt{3} = 1.73$ and $\sqrt{5} = 2.23$)

**(a)** $x^2 - 12 = 0$ **(b)** $x^2 - 21 = 11$ **(c)** $x^2 + 17 = 37$

**3.** From the square of a certain number, if you subtract twice the number, the result is 15. Find the number.

**4.** The product of two consecutive even integers is 120. Find the two integers.

**5.** The length of a rectangular lamina is 3 cm more than the width. If the area is 88 $cm^2$, find the length and the width of the lamina.

**6.** A playing field measuring, 32 metres by 20 metres has a pathway outside the field all around it, of uniform width. The area of the pathway is 285 square metres.

(i) Taking the width of the pathway to be $x$ metres, set up a quadratic equation in $x$ to represent the given information.

(ii) Solve the quadratic equation to find the width of the pathway.

**7.** The hypotenuse of a right angled triangle is $2x + 1$ cm. The lengths of the other two sides are $x$ cm and $x + 7$ cm respectively. Solve for $x$ and find the lengths of all three sides of the triangle.

**8.** The sum of the first $n$ terms of the arithmetic progression $-7, -5, -3, -1, \ldots$ is 105. Use your knowledge on progressions to answer the following.

(i) Taking $n$ to be the number of terms we are adding together, set up a quadratic equation in $n$ for the sum of the first $n$ terms.

(ii) Solve the above equation to find the number of terms $n$ you have added.

## 13.3 Solving a quadratic equation by completing the square

We have seen how to solve a quadratic equation by factorization. This method is useful only when the factorization is easy to do. Many quadratic expressions, such as $x^2 + 3x + 5 = 0$ and $2x^2 - 5x - 1 = 0$ cannot be factorized easily. To solve such quadratic equations we have to use alternative methods. One such method is to arrange the quadratic as a perfect square. This is known as solving the quadratic equation by completing the square.

Before we proceed, let us recall how we learnt to write an expression of the form $x^2 + bx$ as a perfect square.

**Activity**

Write the constant you should include to convert each of the following expressions into a perfect square and write it as a perfect square.

**a.** $x^2 + 6x + 9 \quad = (x + 3)^2$

**b.** $x^2 + 8x + \ldots \quad = \ldots\ldots\ldots\ldots$

**c.** $x^2 - 14x + \ldots = \ldots\ldots\ldots\ldots$

**d.** $x^2 + 3x + \ldots \quad = \ldots\ldots\ldots\ldots$

**e.** $(x + \ldots)^2 = x^2 + 8x + \ldots$

**f.** $(x + \ldots)^2 = x^2 + 2ax + \ldots$

**g.** $(x + b)^2 = x^2 + \ldots x + b^2$

**h.** $(x + m)^2 = x^2 + \ldots x + m^2$

First we will complete the square of a quadratic equation which can also be solved by factoring.

**Example 1**

Solve $x^2 + 2x - 3 = 0$ by completing the square.

First we will keep all terms containing $x$ on one side and move the constant to the right. Then we get

$$x^2 + 2x - 3 = 0$$
$$x^2 + 2x = 3$$

To write the left hand side as a perfect square, take half of the $x$-term coefficient and square it and add this value to both sides. That is, in this problem we have to add +1 to both sides.

$$x^2 + 2x + 1 = 3 + 1$$
$$(x + 1)^2 = 4$$

Take the square root of both sides. We have to allow for both positive and negative values.

$$x + 1 = \pm\sqrt{4}$$
$$x + 1 = \pm 2$$
$$x = \pm 2 - 1$$

Then $x = +2 - 1$ or $x = -2 - 1$

$x = 1$ or $x = -3$

Therefore the solutions to the above equation are $x = 1$ and $x = -3$.

Now let us look at another example.

**Example 2**

Solve $x^2 - 4x + 1 = 0$ by completing the square.

$$x^2 - 4x + 1 = 0$$
$$x^2 - 4x \ - 1$$
$$x^2 - 4x + 4 = -1 + 4$$
$$(x - 2)^2 = 3$$
$$x - 2 = \pm\sqrt{3} \quad \text{(taking the square root of both sides)}$$
$$x = 2 \pm\sqrt{3}$$
$$x = 2 + \sqrt{3} \quad \text{or} \quad x = 2 - \sqrt{3}.$$

Suppose it is given that $\sqrt{3}$ is approximately 1.73

Then $x = 2 + 1.73$ or $x = 2 - 1.73$

$x = 3.73$ or $x = 0.27$

Then the solutions of this equation are $x = 3.73$ and $x = 0.27$.

**Example 3**

Find the roots of $2x^2 + 6x - 5 = 0$ by completing the square.

Here it is easy if we first divide all the terms by 2 to make the leading coefficient (coefficient of the $x^2$ term) one.

$$2x^2 + 6x - 5 = 0$$

$$x^2 + 3x - \frac{5}{2} = 0 \quad \text{(now the leading coefficient is one)}$$

$$x^2 + 3x = \frac{5}{2}$$

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = \frac{5}{2} + \left(\frac{3}{2}\right)^2$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{5}{2} + \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{+10 + 9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{+19}{4}$$

$$x + \frac{3}{2} = \frac{\pm\sqrt{19}}{2}$$

$$x = \frac{+\sqrt{19} - 3}{2} \text{ or } x = \frac{-\sqrt{19} - 3}{2}$$

Suppose 4.36 is given as an approximate value of $\sqrt{19}$.

$$\text{Then} \quad x = \frac{4.36 - 3}{2} \text{ or } x = \frac{-4.36 - 3}{2}$$

$$x = 0.68 \text{ or } x = -3.68$$

The roots of the equation are $x = 0.68$ and $x = -3.68$.

**Exercise13.3**

**1.** Solve the following quadratic equations by completing the square.

(Take $\sqrt{2} = 1.41$, $\sqrt{3} = 1.73$, $\sqrt{5} = 2.23$, $\sqrt{6} = 2.44$, $\sqrt{13} = 3.6$, $\sqrt{17} = 4.12$ and $\sqrt{57} = 7.54$)

**(a)** $x^2 - 2x - 4 = 0$ **(b)** $x^2 + 8x - 2 = 0$ **(c)** $x^2 - 6x = 4$

**(d)** $x^2 + 4x - 8 = 0$ **(e)** $x(x + 8) = 8$ **(f)** $x^2 + x = 4$

**(g)** $2x^2 + 5x = 4$ **(h)** $3x^2 = 3x + \frac{1}{2}$ **(i)** $\frac{2}{x+3} + \frac{1}{2x+3} = 1$

## 13.4 Solving quadratic equations by using the quadratic formula

An easy method of solving a quadratic equation of the form $ax^2 + bx + c = 0$ is by using the quadratic formula.

The quadratic formula is derived by completing the square on a general quadratic equation $ax^2 + bx + c = 0$.

Once the quadratic formula is derived, it is no longer necessary to use the process of completing the square to solve "each" quadratic equation.

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$\frac{ax^2}{a} + \frac{bx}{a} = -\frac{c}{a} \quad \text{( dividing by } a\text{)}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \quad \text{(squaring } \tfrac{1}{2} \text{ of } \tfrac{b}{a} \text{ and adding to both sides)}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

(writing the left hand side as a perfect square and arranging the terms on the right hand side)

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

(simplifying the right hand side by using a common denominator)

Then $$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(simplifying by using a common denominator)

Therefore
to solve a quadratic equation of the form $ax^2 + bx + c = 0$

we can use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

We get two values (roots) for $x$ due to the positive and negative signs .

Here $a$ is the coefficient of $x^2$

$b$ is the coefficient of $x$ and

$c$ is the constant term.

**Example 1**

Solve $2x^2 + 7x + 3 = 0$ by using the quadratic formula.

To solve the equation $2x^2 + 7x + 3 = 0$, we can take $a = 2$, $b = 7$ and $c = 3$ and substitute these values in the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 2 \times 3}}{2 \times 2}$$

$$= \frac{-7 \pm \sqrt{49 - 24}}{4}$$

$$= \frac{-7 \pm \sqrt{25}}{4}$$

$$= \frac{-7 \pm 5}{4}$$

$$x = \frac{-7 + 5}{4} \quad \text{or } x = \frac{-7 - 5}{4}$$

$$x = -\frac{1}{2} \text{ or } x = -3$$

$x = -\dfrac{1}{2}$ and $x = -3$ are the solutions of the above equation.

**Example 2**

Solve $4x^2 - 7x + 2 = 0$ by using the quadratic formula, and find its roots

Take $\sqrt{17} = 4.12$.

$$4x^2 - 7x + 2 = 0$$

Substitute $a = 4$, $b = -7$, $c = 2$. (According to the equation $ax^2 + bx + c = 0$)

$$\text{in } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 4 \times 2}}{2 \times 4}$$

$$= \frac{7 \pm \sqrt{49 - 32}}{8}$$

$$= \frac{7 \pm \sqrt{17}}{8}$$

$$= \frac{7 \pm 4.12}{8} \quad (\sqrt{17} = 4.12 \text{ is given})$$

$$x = \frac{7 + 4.12}{8} \quad \text{or} \quad x = \frac{7 - 4.12}{8}$$

$$x = \frac{11.12}{8} \quad \text{or} \quad x = \frac{2.88}{8}$$

$x = 1.39$ or $x = 0.36$

$x = 1.39$ and $x = 0.36$ are the roots of the equation.

**Example 3**

Solve $x^2 + 2x - 1 = 0$ by using the quadratic formula and find the roots accurate to the second decimal place.

(Take $\sqrt{2} = 1.414$).

$a = 1, b = 2, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$= \frac{-2 \pm \sqrt{4 + 4}}{2}$$

$$= \frac{-2 \pm \sqrt{8}}{2}$$

$$= \frac{-2 \pm \sqrt{4 \times 2}}{2}$$

$$= \frac{-2 \pm 2\sqrt{2}}{2}$$

$$= \frac{-2 \pm 2 \times 1.414}{2}$$

$$= \frac{-2 \pm 2.828}{2}$$

$$x = \frac{-2 + 2.828}{2} \quad \text{or} \quad x = \frac{-2 - 2.828}{2}$$

$$= \frac{0.828}{2} \qquad x = \frac{-4.828}{2}$$

$$x = 0.414 \text{ or } x = -2.414$$

The roots are $x = 0.41$ and $x = -2.41$.

## Exercise13.4

1. Solve the following quadratic equations using the quadratic formula and give your answers accurate to the first decimal place.

   (Take $\sqrt{3} = 1.73$, $\sqrt{17} = 4.12$ and $\sqrt{29} = 5.38$)

   **(a)** $x^2 - 6x - 3 = 0$ **(b)** $x^2 - 7x + 5 = 0$ **(c)** $2x^2 - x - 2 = 0$

   **(d)** $2x^2 - 5x + 1 = 0$ **(e)** $3x^2 - 4x - 7 = 0$

## Miscellaneous Exercise

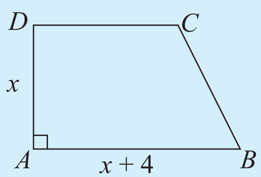
1. When three times a number which is postive is subtracted from the square of that number the answer is 28. Find the number.

2. The product of two consecutive odd integers is 99. Find the two integers.

3. The area of a rectangular sheet is 44 square centimetres. The length of the sheet is 6 centimetres more than the width. Let $x$ be the width of the sheet in centimetres.

   (i) Set up a quadratic equation in $x$ to represent the given information.

   (ii) Solve the equation to find the value of $x$ accurate to the first decimal place. (Take $\sqrt{53} = 7.28$). Hence find the length of the sheet.

4. 


   $ABCD$ is a trapezium with $AD = CD$.

   (i) If the area of the trapezium is 12 cm$^2$ show that $x$ satisfies $x^2 + 2x - 12 = 0$.

   (ii) Solve the quadratic equation in (i) above by completing the square or by some other method and find the value of $x$ to the nearest first decimal place.

5. The squares of three consecutive natural numbers add up to 149.Taking $x$ to be the number in the middle, find the largest number.

6. In a right angled triangle the lengths of the two sides containing the right angle are $5x$ cm and $(3x - 1)$ cm. If the area is given as 60 square centimetres, set up a quadratic equation in $x$ and solve for $x$. Thereby find the length of each side of the triangle.

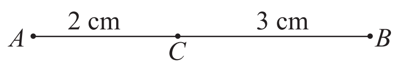
7. A man bought a certain number of mangoes for Rs 600.If the price of a mango was one rupee less, then he could have bought 20 more mangoes. Find the number of mangoes he bought.

# 14 Equiangular Triangles

**By studying this lesson you will be able to**

- understand the meaning of equiangular and similar figures,
- identify the theorem "A line drawn parallel to a side of a triangle, divides the other two sides proportionally",
- identify the converse theorem "If a straight line divides two sides of a triangle proportionally then that line is parallel to the remaining side",
- identify the theorem "Corresponding sides of equiangular triangles are proportional",
- identify the converse theorem "If the corresponding sides of two triangles are proportional then those triangles are equiangular".

## Ratios of lengths

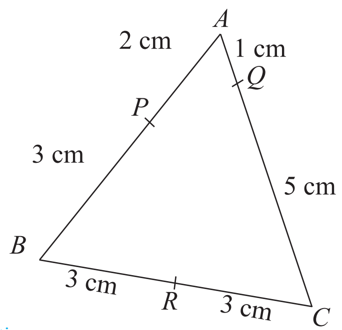



The figure illustrates the line segment $AB$. The point $C$ lies on $AB$ such that $AC = 2$ cm and $CB = 3$ cm. Point $C$ divides $AB$ into two line segments $AC$ and $CB$. Then, the ratio of $AC$ to $CB$ can be written using their lengths as follows.

| | | |
|---|---|---|
| | $AC : CB = 2 : 3$ | |
| Similarly | $AC : AB = 2 : 5$ | (since $AB = 5$cm) |
| hence | $CB : AC = 3 : 2$ | |
| | $CB : AB = 3 : 5.$ | |

The ratio of the lengths of line segments should be written in the order of the line segments relevant to that ratio.

Consider the triangle $ABC$ in the following figure.



As shown in the figure, the points $P$, $Q$ and $R$ lie on each side of the triangle $ABC$. Then the ratios can be written as follows.

(i) $AP : PB = 2 : 3$, $AP : AB = 2 : 5$, $PB : AP = 3 : 2$
(ii) $AQ : QC = 1 : 5$, $AQ : AC = 1 : 6$, $QC : AQ = 5 : 1$
(iii) $BR : RC = 3 : 3 = 1 : 1$, $BR : BC = 3 : 6 = 1 : 2$

We have already learnt that ratios can be written as fractions.

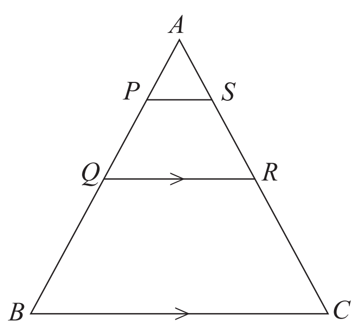
Therefore $AQ : QC = 1 : 5$ can be also written as $\frac{AQ}{QC} = \frac{1}{5} = 0.2$

## 14.1 Dividing two sides of a triangle by a line drawn parallel to the other side

Let us do the following activity to find out about the ratios in which two sides of a triangle are divided by a line drawn parallel to the other side.

**Activity**

- Draw a triangle $ABC$ with $AB$ = 6cm and choosing any length for the other two sides.
- Mark the points $P$ and $Q$ on $AB$ such that $AP$ = 2 cm and $AQ$ = 3 cm.
- Using the set square or using any other method, draw a line through $Q$ parallel to $BC$ and name the point that it meets the line $AC$ as $R$.




- Measure $AR$ and $RC$.
- Similarly draw another line parallel to $BC$ through $P$ and mark the point that it meets the line $AC$ as $S$.
- Measure $AS$ and $SC$.
- Now complete the following table.

| State | Ratio between segments of $AB$ | Ratio between segments of $AC$ | Relationship between the two ratios |
|---|---|---|---|
| Parallel line through $Q$ | $\frac{AQ}{QB} = \frac{3}{3} = 1$ | $\frac{AR}{RC} =$ | |
| Parallel line through $P$ | $\frac{AP}{PB} = \frac{2}{4} = 0.5$ | $\frac{AS}{SC} =$ | |

- Similarly check the relationship between the ratios in which the two sides of a right angled triangle and obtuse angled triangle are divided by a line drawn parallel to the other side.

Check whether your results agree with the following sentence.

**A line drawn parallel to one side of a triangle divides the other two sides in equal ratios.**

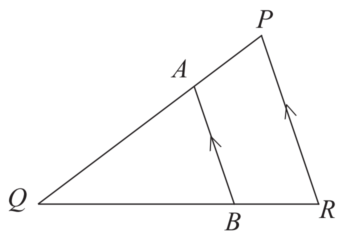
The above result can be given as a theorem in geometry.

**Theorem:**

**A line drawn parallel to a side of a triangle divides the other two sides proportionally.**

As an example, the line $AB$ drawn parallel to the side $PR$ of the triangle $PQR$ is shown in the figure.
Then according to the theorem,

(i) $QA : AP = QB : BR$ hence, $\frac{QA}{AP} = \frac{QB}{BR}$.



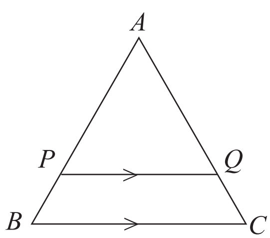



Figure (i)

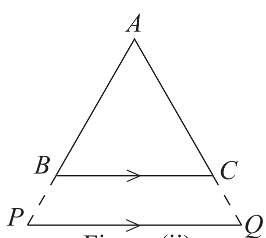


Figure (ii)

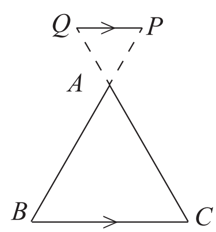


Figure (iii)

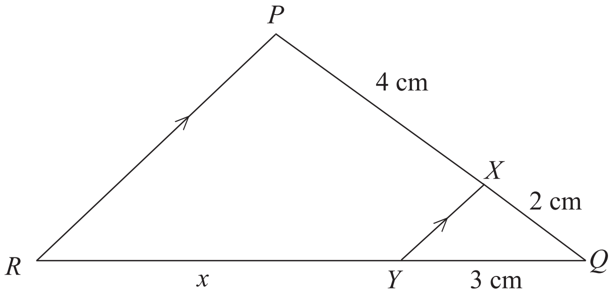
As shown in figure (i), the line $PQ$ drawn parallel to $BC$ internally divides the sides $AB$ and $AC$. But in figure (ii) and figure (iii), the line $PQ$, drawn parallel to $BC$ meets other two sides produced at $P$ and $Q$. In these kinds of situations, it is said that $PQ$ externally intersects $AB$ and $BC$. Irrespective of whether the sides are divided internally or externally, the theorem is valid.

Hence for all three figures above, $\frac{AP}{PB} = \frac{AQ}{QC}$.

Now consider the following examples with calculations using the theorem.

**Example 1**

In triangle $PQR$, the line $XY$ is drawn parallel to $PR$. Find the length of $RY$ if $PX = 4$ cm, $XQ = 2$ cm, and $YQ = 3$ cm.




Let $x$ be the length of $RY$.
Then, since $XY$ is drawn parallel to $PR$,

according to the theorem $\frac{RY}{YQ} = \frac{PX}{XQ}$

hence $\frac{x}{3} = \frac{4}{2}$

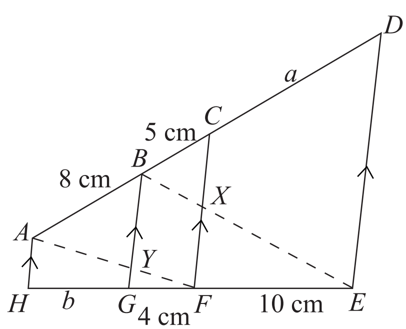
$\therefore \quad 2x = 4 \times 3$

$\therefore \quad x = 6$

$\therefore$ The length of $RY$ is 6 cm.

**Example 2**

Based on the information given in the figure, find the values of $a$ and $b$.



Let us join *BE* first.

In triangle *BED,* as *DE//CX*, according to the theorem, *CX* divides the sides *BD* and *BE* proportionally.

Hence, $\frac{BC}{CD} = \frac{BX}{XE}$

Hence, $\frac{5}{a} = \frac{BX}{XE}$ ——① 

Now, in triangle *BGE*, as *BG//XF*, according to the theorem, *XF* divides the sides *EB* and *EG* proportionally.

Hence,

$$\frac{BX}{XE} = \frac{GF}{FE}$$

$$\therefore \quad \frac{BX}{XE} = \frac{4}{10} \quad \text{——②}$$

From ① and ②

$$\frac{5}{a} = \frac{4}{10}$$

Hence, $4a = 50$

$$\therefore \quad a = \frac{50}{4}$$

$$= 12.5 \text{ cm}$$

Similarly, let us join *AF.*

In triangle *ACF*, $\frac{AB}{BC} = \frac{AY}{YF}$

$$\frac{8}{5} = \frac{AY}{YF} \quad \text{——③}$$

In triangle *AHF*, $\frac{AY}{YF} = \frac{HG}{GF}$

$$\frac{AY}{YF} = \frac{b}{4} \quad \text{——④}$$

From ③ and ④,

$$\frac{b}{4} = \frac{8}{5}$$

$$\text{hence } 5b = 32$$

$$\therefore \quad b = \frac{32}{5}$$

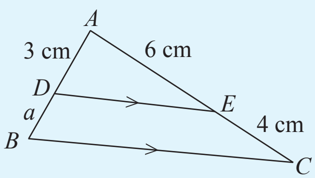
$$= 6.4 \text{ cm}$$

Now establish what was learnt by doing the calculations in the following exercise.

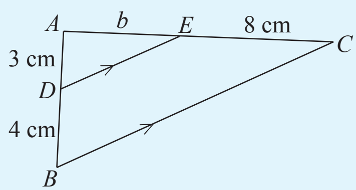
## Exercise14.1

**1.** In each of the following figures, the lengths of some line segments are denoted by unknown terms. Find the values of those unknown terms.

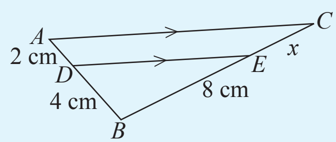
(i)



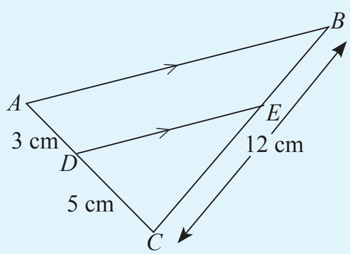

(ii)



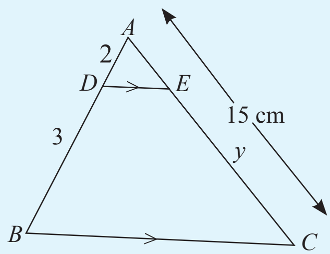

(iii)



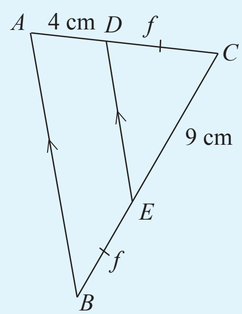

(iv)



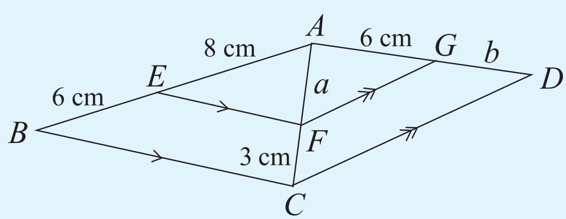

(v)



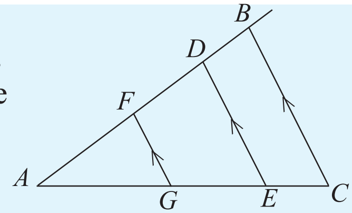

(vi)



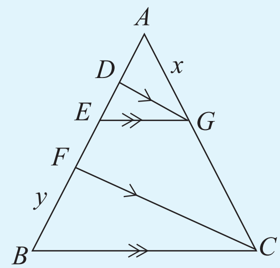

**2.** Based on the information in the figure, find the values of $a$ and $b$.



**3.** In the given figure, $FG//DE//BC$. Also, $AF = 6$ cm, $DB = 3$ cm, $AG = 8$ cm and $GE = 8$ cm. Find the lengths of the line segments $FD$ and $EC$.




**4.** In the given figure, $DG//FC$ and $EG//BC$ . $AD = 6$ cm, $DE = 4$ cm, $EF = 5$ cm and $GC = 18$ cm. Find the values of $x$ and $y$.

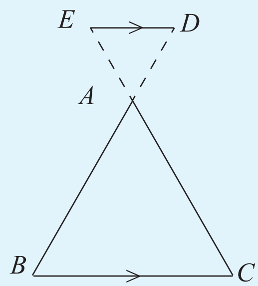



**5.** As shown in the figure, in triangle $ABC$, the line $ED$ drawn parallel to $BC$ externally divides the sides $BA$ and $CA$ produced. Also $AE = 2$ cm, $AD = 3$ cm and $AC = 4$ cm. The length of the line segment $AB$ is denoted by $x$.

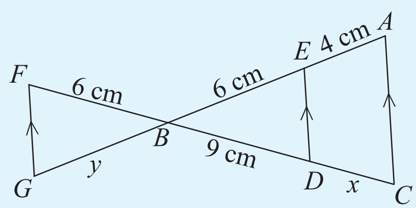
(i) Fill in the blanks.

$DB$ : .... = .... : $EA$

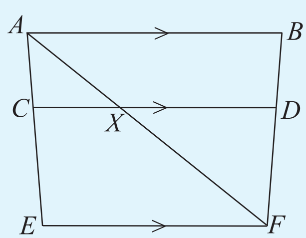
(ii) Find the value of $x$.




**6.** Based on the given information in the figure, find the values of $x$ and $y$.




**7.** In the given figure $AB//CD//EF$. $AC = 3$ cm, $CE = 5$ cm and $BF = 12$ cm. Find the lengths of $BD$ and $DF$.



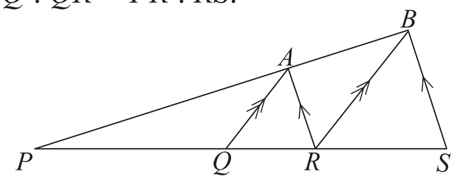

**8.** In a triangle $ABC$, the bisector of $B\hat{C}A$ meets $AB$ at $X$. The point $P$ lies on $BC$ such that $PX = PC$. If $PX = 9$ cm, $BX = 5$ cm and $AX = 6$ cm, then find the length of $BC$.

## 14.2 More on dividing two sides of a triangle proportionally

In this section let us consider how riders are proved using the theorem "A line drawn parallel to a side of a triangle divides the other two sides proportionally"

**Example 1**

In the given figure, $PQRS$ and $PAB$ are straight lines. $BS // AR$ and $BR // AQ$.
Prove that $PQ : QR = PR : RS$.




**Proof:** In the triangle $PBR$, as $AQ$ is parallel to $BR$, according to the theorem,

$PA : AB = PQ : QR$ ——① 

In the triangle $PBS$ as $AR$ is parallel to $BS$, according to the theorem

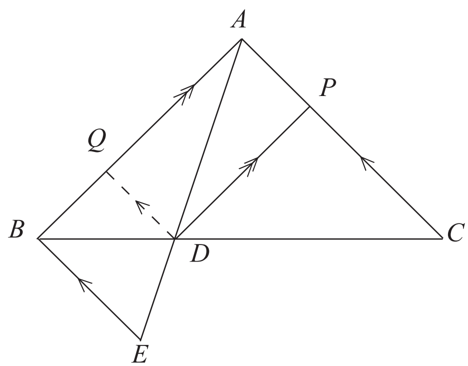
$PA : AB = PR : RS$ ——② 

From ① and ②

$PQ : QR = PR : RS$

**Example 2**

The point $D$ lies on the side $BC$ of the triangle $ABC$. The line $BE$ drawn parallel to $AC$ meets $AD$ produced at $E$. The line drawn from $D$, parallel to $AB$ meets $AC$ at $P$. Prove that $CP : AP = AD : DE$.




Here, as in the above example, choose two triangles and in each triangle draw a line parallel to a relevant side of the triangle. We choose triangles $ABE$ and $ABC$, because there is a common side to these two triangles.

But there is no line parallel to a side of the $ABC$ triangle. Therefore let us construct such a line.

**Construction**: Draw line $DQ$ parallel to $BE$ such that it meets $AB$ at $Q$. (Then $AC$, $QD$, and $BE$ are parallel to each other)

**Proof** :

In the triangle $ABC$, $PD$ is parallel to $AB$. Therefore by the theorem,

$$CP : PA = CD : DB \quad \text{——} ①$$

In the triangle $ABC$, $QD$ is parallel to $AC$. Therefore by the theorem,

$$AQ : QB = CD : DB \quad \text{——} ②$$

In the triangle $ABE$, $QD$ is parallel to $BE$. Therefore by the theorem,

$$AQ : QB = AD : DE \quad \text{——} ③$$

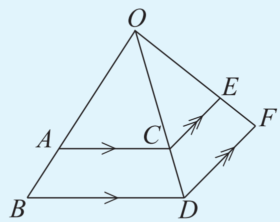
From the equations ①, ② and ③

$$CP : PA = CD : DB = AQ : QB = AD : DE.$$

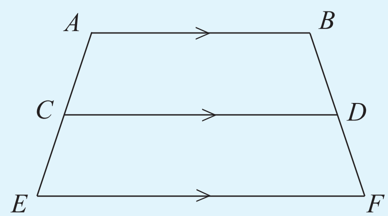
$$\therefore \quad CP : PA = AD : DE$$

## Exercise 14.2

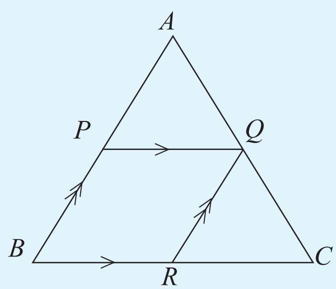
**1.** Based on the information in the figure, show that $OA : AB = OE : EF$.




**2.** Based on the given information in the figure, prove that $AC : CE = BD : DF$.

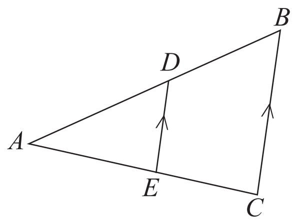



**3.** Based on the given information in the figure, prove that $AP : AB = BR : BC$.



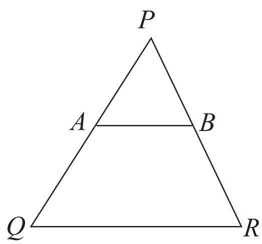

**4.** In triangle $PQR$, the point $A$ lies on $QR$. A line drawn through $A$, parallel to $PR$, meets $PQ$ at $B$. The line $RCD$ drawn from $R$, intersects $AB$ at $C$ and $PQ$ at $D$. If $D\hat{B}C = B\hat{C}D$ then prove that $\dfrac{QA}{AR} = \dfrac{QB}{CR}$ .

## 14.3 The converse of the theorem "a line drawn parallel to a side of a triangle divides the other two sides proportionally "




The above theorem says that, in triangle $ABC$, a line drawn parallel to $BC$ divides $AB$ and $AC$ in equal ratios.

That is, since $BC//DE$, $AD : DB = AE : EC$. Let us understand the converse of this theorem by considering the triangle $PQR$ shown in the figure.



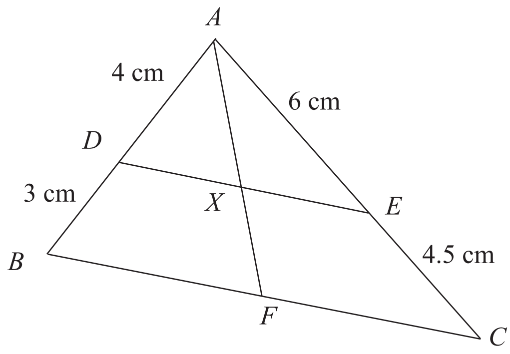

Here the line $AB$ intersects the sides $PQ$ and $PR$. The ratios of the line segments of each side are $PA : AQ$ and $PB : BR$.

If these two ratios are equal, that is, if $PA : AQ = PB : BR$ then the line $AB$, which intersects those two sides at $P$ and $Q$, is parallel to the other side $QR$. This is the converse of the theorem we have already learnt in this lesson. This result can be stated as a theorem.

**The converse of the above theorem:**
**If a line divides two sides of a triangle proportionally, then that line is parallel to the other side.**

Given below are some examples with calculations and proved riders related to the theorem.

**Example 1**




Based on the information in the figure, find the value of $AX : XF$.

Consider the triangle $ABC$. Then, $AD : DB = 4 : 3$ and

$$AE : EC = 6 : 4.5 = 4 : 3.$$

$$\therefore \quad AD : DB = AE : EC$$

$\therefore$ The line $DE$ divides $AB$ and $AC$ proportionally.

$\therefore$ By the converse of the theorem, $DE//BC$.

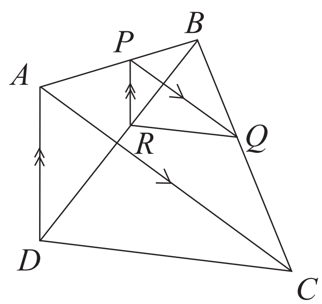
Therefore in triangle $ABF$, since $DX//BF$,

$AD : DB = AX : XF$

But $AD : DB = 4 : 3$

Hence $AX : XF = \underline{\underline{4 : 3}}$

**Example 2**




The point $P$ lies on the side $AB$ of the quadrilateral $ABCD$. The line drawn through $P$, parallel to $AC$, meets $BC$ at $Q$. Similarly, the line drawn through $P$, parallel to $AD$, meets $BD$ at $R$. Prove that $RQ//DC$.

**Proof** :

In triangle $ABD$, since $PR$ is parallel to the side $AD$,

$BP : PA = BR : RD$ ——① 

In triangle $ABC$, since $PQ$ is parallel to the side $AC$,

$BP : PA = BQ : QC$ ——②

From equations ① and ②

$BR : RD = BQ : QC$.

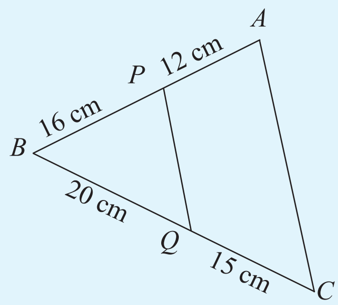
$\therefore$ In triangle $BDC$, the line $RQ$ divides the sides $BD$ and $BC$ proportionally.

$\therefore$ $RQ//DC$ (by the converse theorem)

For the following exercises use the converse theorem given above.

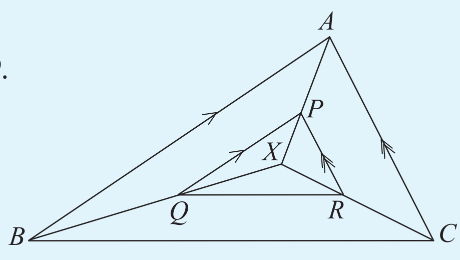
## Exercise 14.3

**1.** Based on the information given in the figure, show that $AC$ is parallel to $PQ$.

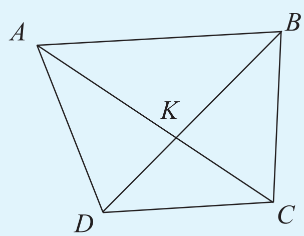



**2.** In triangle $ABC$, the point $P$ lies on the side $AB$ and the point $Q$ lies on the side $AC$ such that $AP : PB = AQ : QC$. Prove that $Q\hat{P}B + P\hat{B}C = 180^\circ$.

**3.** In the given figure, $AC//PR$ and $AB//PQ$. Prove that $BC//QR$.

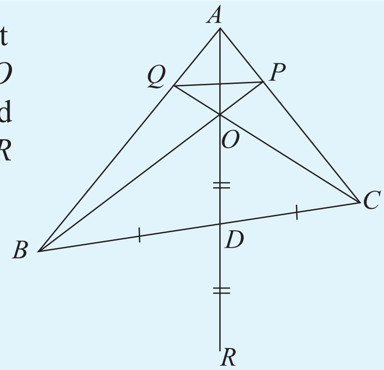



**4.** In the quadrilateral $ABCD$ given in the figure, the diagonals $AC$ and $BD$ intersect at $K$. If $AK = 4.8$ cm, $KC = 3.2$ cm, $BK = 3$ cm and $KD = 2$ cm, then show that $DC$ is parallel to $AB$.(Hint: In triangle $KDC$, take that the points $A$ and $B$ lie on $DK$ and $CK$ produced respectively.)



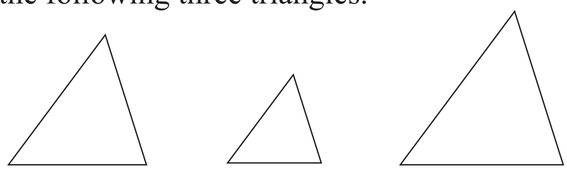
**5.** In the triangle $ABC$ given in the figure, the midpoint of the side $BC$ is $D$. The point $O$ lies on $AD$. $BO$ produced intersects $AC$ at $P$ and $CO$ produced intersects $AB$ at $Q$. The line $AD$ is produced to $R$ such that $OD = DR$. Prove that,

(i) $BRCO$ is a parallelogram.
(ii) $AQ : QB = AO : OR$.
(iii) $QP // BC$.

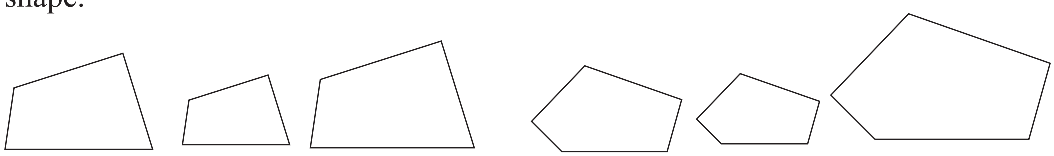



## 14.4 Similar figures and equiangular figures

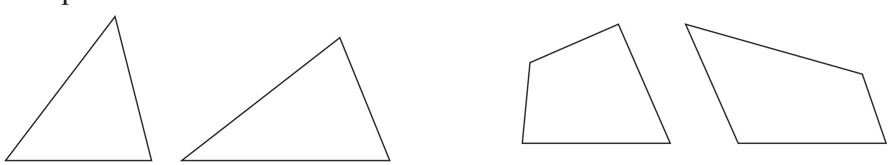
Carefully observe the following three triangles.



In day to day language we say that these three triangles are of the "same shape". The following figure illustrates three quadrilaterals and three pentagons of the same shape.



But the following pair of triangles as well as pair of quadrilaterals are not of the same shape.

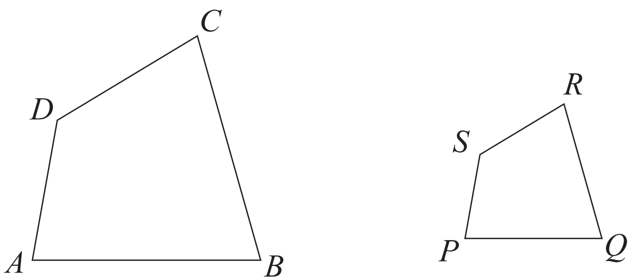


Did you think of what is meant by "shape" here? In mathematics things should be defined precisely. It is necessary to give a precise definition for "shape". "Similar figure" is the phrase in mathematics equivalent to "same shape" in day to day language. Here we consider only the similarity of polygons.

Two polygons are said to be similar if;

1. the angles of one polygon are equal to the angles of the other and,
2. the corresponding sides of the polygons are proportional.

As an example, consider the following two quadrilaterals *ABCD* and *PQRS*.




In these two quadrilaterals,

if $\hat{A} = \hat{P}, \hat{B} = \hat{Q}, \hat{C} = \hat{R}, \hat{D} = \hat{S}$ and

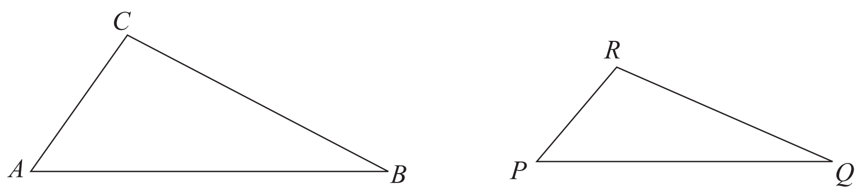
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DA}{SP},$$

then the quadrilaterals *ABCD* and *PQRS* are similar.

In this lesson we hope to study more about similar triangles.
In the two triangles *ABC* and *PQR* given below,

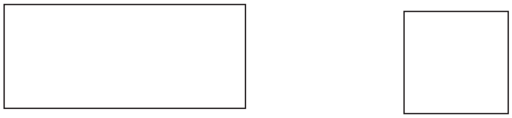
if $\hat{A} = \hat{P}, \hat{B} = \hat{Q}, \hat{C} = \hat{R}$

and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$, then by definition the two triangles are similar.




There is an important result related to the similarity of triangles. That is, if the angles of one triangle are equal to the angles of another triangle, then those two triangles are similar. In otherwords, if three angles of a triangle are equal to the three angles of another triangle then the corresponding sides of those two triangles are proportional. Therefore to find out whether two triangles are similar, it is sufficient to check only the angles of the two triangles. As an example, in the above two triangles, if $\hat{A} = \hat{P}, \hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}$, then $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$.

This result is not true for polygons which are not triangles. For example, in the following two quadrilaterals, the angles of one quadrilateral are equal to the angles of the other quadrilateral. Each of these angles is $90^\circ$. However one of the quadrilaterals is a rectangle and the other is a square. Therefore their sides are not proportional and hence the two quadrilaterals are not similar.



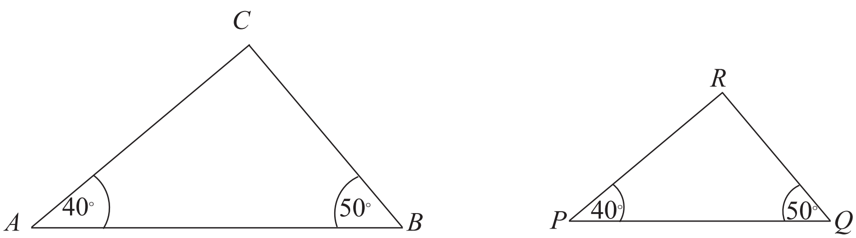
If the angles of two polygons are equal, then those polygons are said to be equiangular. According to the above discussion, two equiangular triangles are similar too. Let us use this result as a theorem without proof.

**Theorem on equiangular triangles:**
**If two triangles are equiangular then the corresponding sides are proportional.**

Do the following activity to understand more about this result.

**Activity**

- Using the protractor, draw two different sized triangles, each with interior angles $40^\circ$, $50^\circ$ and $90^\circ$. As shown below, name the triangles as $ABC$ and $PQR$.




- Find the ratios(as fractions) between the corresponding sides of the two triangles. That is, find the values of $\frac{AB}{PQ}$, $\frac{BC}{QR}$ and $\frac{CA}{RP}$ separately.
- Check whether these three values are equal.(You can have minor errors because of errors in the measurements)

According to the above activity, you can understand that if two triangles are equiangular then they are similar.

**Note:**

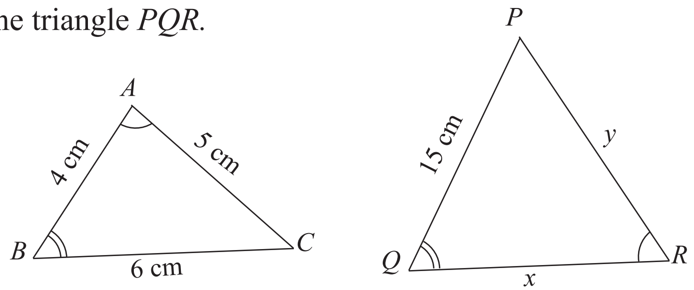
1.If restricted to triangles, similar and equi-angular mean the same thing.

2.It is clear that two congruent triangles are similar. But two similar triangles may not be congruent.

3. If two angles of one triangle are equal to two angles of another triangle, then the remaining pair of angles are also equal to each other. The reason for this is because the sum of the three angles of a triangle is equal to 180°. Therefore, if two angles of one triangle are equal to two angles of another triangle, then they are equiangular.

**Example 1**

In the two triangles $ABC$ and $PQR$ in the figure, $\hat{A} = \hat{R}$ and $\hat{B} = \hat{Q}$. Find the values of $x$ and $y$ in the triangle $PQR$.




In the two triangles $ABC$ and $PQR$

$\hat{A} = \hat{R}$ and $\hat{B} = \hat{Q}$

$\therefore$ $\hat{C} = \hat{P}$ (since the sum of the three interior angles of a triangle is 180°)

$\therefore$ The triangles $ABC$ and $PQR$ are equiangular.

$\therefore$ Corresponding sides are propotional.

Hence; $\dfrac{BC}{PQ} = \dfrac{AB}{QR}$

$\therefore \quad \dfrac{6}{15} = \dfrac{4}{x}$

$6x = 15 \times 4$ (By cross multiplication)

$\therefore \quad x = \dfrac{15 \times 4}{6}$

$= \underline{\underline{10 \text{ cm}}}$

$\dfrac{BC}{PQ} = \dfrac{AC}{PR}$

$\dfrac{6}{15} = \dfrac{5}{y}$

$6y = 15 \times 5$

$y = \dfrac{15 \times 5}{6}$

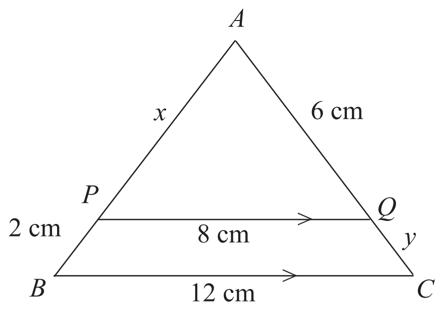
$= \underline{\underline{12.5 \text{ cm}}}$

**Example 2**

In the triangle $ABC$, the line $PQ$ is drawn parallel to the side $BC$.

(i) Show that the triangles $ABC$ and $APQ$ are equiangular.

(ii) Find the values of $x$ and $y$.




(i) In triangles $ABC$ and $APQ$,

$A\hat{B}C = A\hat{P}Q$ (corresponding angles, $BC//PQ$)

$A\hat{C}B = A\hat{Q}P$ (corresponding angles, $BC//PQ$)

$\hat{A}$ is common to both triangles

$\therefore$ The triangles $ABC$ and $APQ$ are equiangular.

(ii) Since $ABC$ and $APQ$ are equiangular triangles, according to the theorem, the corresponding sides are proportional.

$$\therefore \frac{BC}{PQ} = \frac{AB}{AP}$$

$$\therefore \frac{12}{8} = \frac{x+2}{x}$$

$$12x = 8(x+2)$$

$$12x = 8x + 16$$

$$12x - 8x = 16$$

$$4x = 16$$

$$\underline{\underline{x = 4 \text{ cm}}}$$

$$\therefore \frac{BC}{PQ} = \frac{AC}{AQ}$$

$$\therefore \frac{12}{8} = \frac{6+y}{6}$$

$$8(6+y) = 6 \times 12$$

$$48 + 8y = 72$$

$$8y = 72 - 48$$

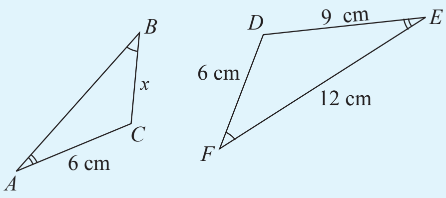
$$8y = 24$$

$$\underline{\underline{y = 3 \text{ cm}}}$$

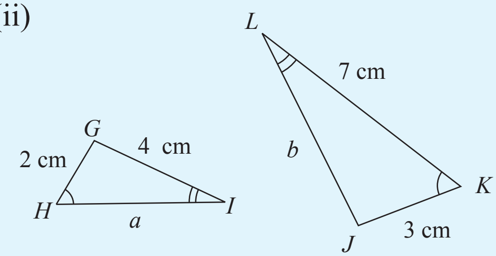
## Exercise14.4

**1.** For each pair of triangles given below, find the lengths of the sides represented by unknowns.

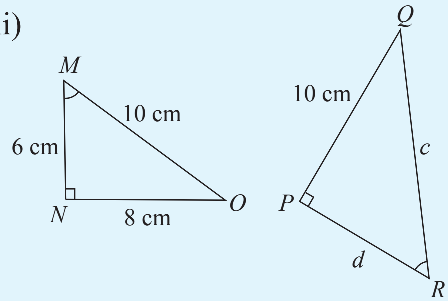
(i)



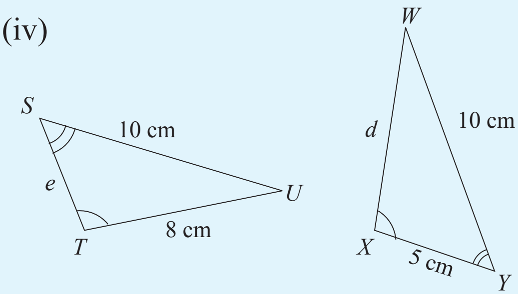

(ii)




(iii)

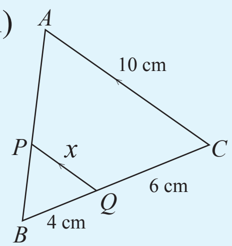



(iv)

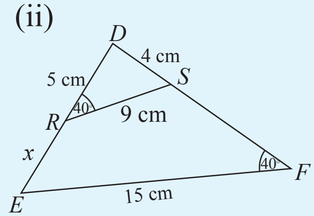



**2.** Show that each pair of triangles in each of the following figures is equiangular and find the lengths of sides represented by unknowns.

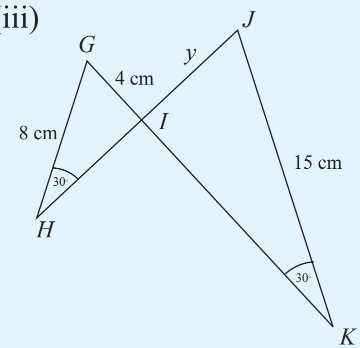
(i)




(ii)




(iii)

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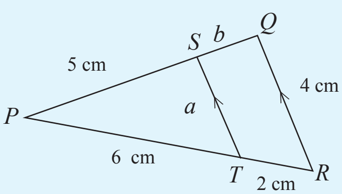
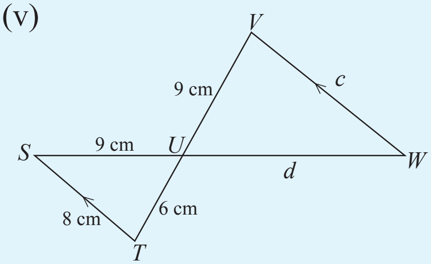


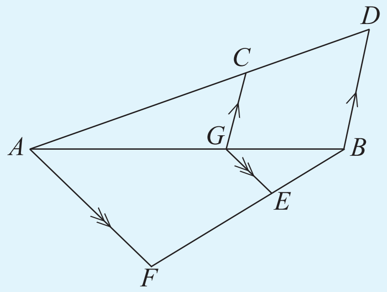

(v)



3. Based on the information given in the figure

(i) name two pairs of equiangular triangles.

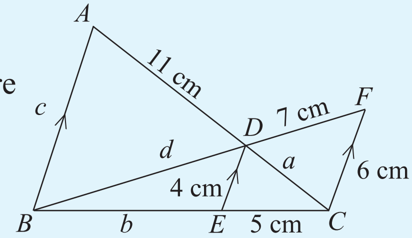
(ii) if $BD = 9$ cm, $GC = 6$ cm, $AG = 12$ cm and $GE = 2$ cm, then find the lengths of $GB$ and $AF$.




4. According to the information given in the figure

(i) name three pairs of equiangular triangles.

(ii) find the lengths of the sides represented by $a$, $b$, $c$ and $d$.




Next we investigate the converse of the above theorem. That is, to find whether the statement "if the sides of two triangles are proportional then those triangles are equiangular" is true .This converse is true and we can state this result as a theorem.

Also, if the three sides of a triangle are proportional to the three sides of another triangle then the two triangles are said to be similar.

Do the following activity to understand this result further.

**Activity**

- Construct the triangle $ABC$ with $AB = 2.5$ cm, $BC = 3$ cm and $AC = 3.5$ cm.
- Construct the triangle $PQR$ with $PQ = 5$ cm, $QR = 6$ cm and $PR = 7$ cm.
- Observe the relationship between the values $\frac{AB}{PQ}, \frac{BC}{QR}, \frac{AC}{PR}$ .
- Measure the angles of each triangles separately.
- What type of triangles are $ABC$ and $PQR$?

Through this activity you may have observed that the corresponding sides of the two triangles are proportional and also that the angles of $ABC$ are equal to the angles of $PQR$.

This result can be expressed as the converse theorem of the theorem which we learnt about equiangular triangles earlier.

**Theorem: If the three sides of a triangle are proportional to the three sides of another triangle, then the two triangles are equiangular.**

**Example 1**

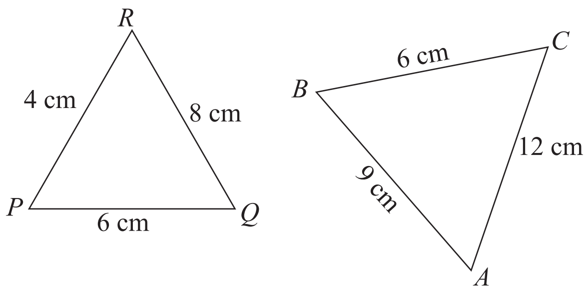
According to the lengths of the sides given in the figure, show with reasons that the triangles $ABC$ and $PQR$ are equiangular. Name the equal angles.

By writing the ratios of the sides according to the given lengths,

(i) $\frac{PQ}{AB} = \frac{6}{9} = \frac{2}{3}$

(ii) $\frac{RQ}{CA} = \frac{8}{12} = \frac{2}{3}$

(iii) $\frac{PR}{BC} = \frac{4}{6} = \frac{2}{3}$




Since these ratios are equal, according to the converse theorem, the triangles $PQR$ and $ABC$ are equiangular.

In the triangle $PQR$, $\hat{R}$ is the angle opposite $PQ$

$\hat{Q}$ is the angle opposite $PR$

$\hat{P}$ is the angle opposite $QR$

In the triangle $ABC$, $\hat{C}$ is the angle opposite $AB$

$\hat{A}$ is the angle opposite $BC$

$\hat{B}$ is the angle opposite $AC$

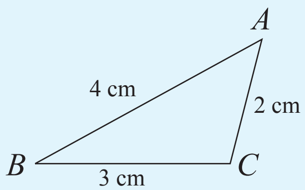
$\therefore \hat{P} = \hat{B}, \hat{Q} = \hat{A}, \hat{R} = \hat{C}$

Do the following exercise using the converse theorem " if corresponding sides are proportional then the triangles are equiangular"

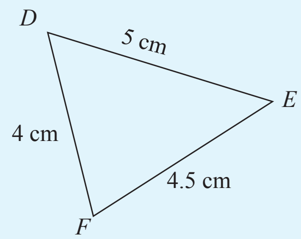
## Exercise 14.5

**1.** From the sketched triangles with measurements given below, choose three pairs of equiangular triangles.

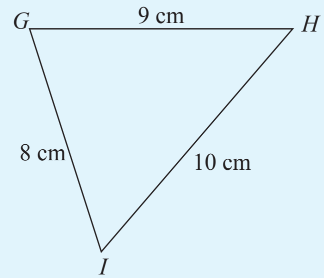
(i)



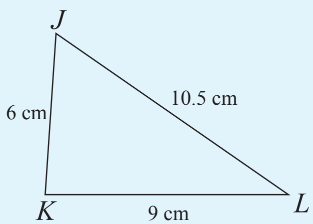

(ii)



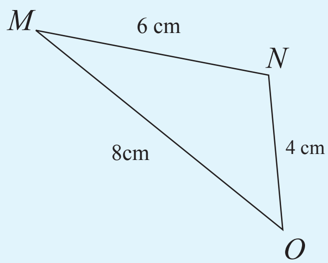

(iii)



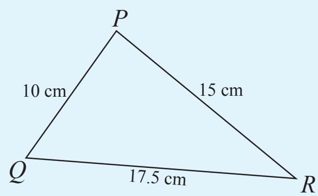

(iv)



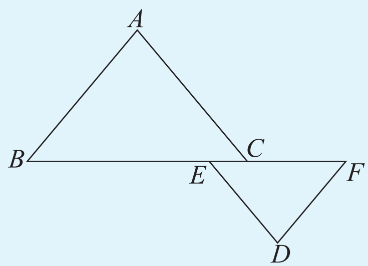

(v)



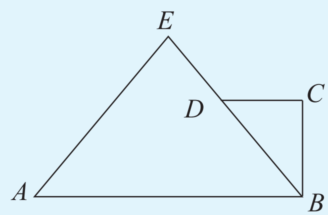

(vi)




**2.** In the given figure, $\frac{AB}{EF}=\frac{AC}{ED}=\frac{BC}{DF}$. Name an angle which is equal to each of $B\hat{A}C$, $A\hat{B}C$ and $A\hat{C}B$.



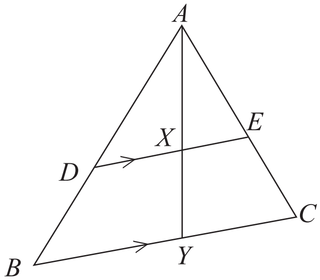

**3.** In the given figure, $AB = 20$ cm, $BC = 6$ cm, $CD = 4$ cm, $DB = 8$ cm, $DE = 2$ cm and $AE = 15$ cm. $AB//DC$. Also $CD$ produced meets $AE$ at $F$. Find the length of $AF$.



## 14.5 Proving riders using the theorem of equiangular triangles

Let us learn how to prove riders by using the theorems learnt so far appropriately. For that study the following example.

**Example 1**




In triangle $ABC$, the points $D$ and $E$ lie on the sides $AB$ and $AC$ such that $DE//BC$. The line $AY$ cuts $DE$ at $X$ and $BC$ at $Y$.

Prove that

(i) $\dfrac{XE}{YC} = \dfrac{AX}{AY}$

(ii) $\dfrac{XE}{YC} = \dfrac{DX}{BY}$

**Proof** : (i) In triangles, $AXE$ and $AYC$ in the figure ;

$A\hat{X}E = A\hat{Y}C$ (corresponding angles, $XE//YC$)

$A\hat{E}X = A\hat{C}Y$ (corresponding angles, $XE//YC$)

$\hat{A}$ is common to both triangles.

$\therefore$ $AXE$ and $AYC$ are equiangular triangles.

$\therefore$ corresponding sides are proportional.

Then; $\dfrac{AX}{AY} = \dfrac{XE}{YC}$ (By theorem)

(ii) In triangles, $ADX$ and $ABY$ in the figure,

$A\hat{D}X = A\hat{B}Y$ (corresponding angles, $DX//BY$)

$A\hat{X}D = A\hat{Y}B$ (corresponding angles, $DX//BY$)

$\hat{A}$ is common to both triangles.

$\therefore$ $ADX$ and $ABY$ are equiangular triangles.

$\therefore$ corresponding sides are proportional.

$\therefore \quad \dfrac{AX}{AY} = \dfrac{DX}{BY}$

But $\dfrac{AX}{AY} = \dfrac{XE}{YC}$ (proved)

$\therefore \quad \dfrac{XE}{YC} = \dfrac{DX}{BY}$

Now do the following exercise.

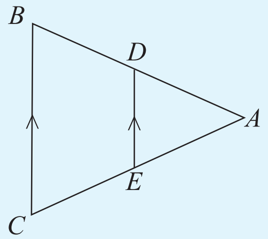
**Exercise14.6**

**1.** Based on the information given in the figure,

(i) show that triangles $ADE$ and $ABC$ are equiangular.

(ii) prove that $\dfrac{AD}{AB} = \dfrac{DE}{BC}$.

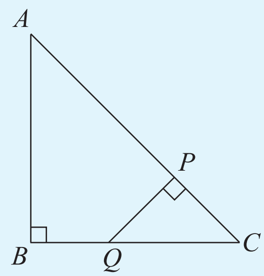
(iii) prove that $\dfrac{AE}{ED} = \dfrac{AC}{CB}$.




**2.** Based on the information given in the figure, prove that

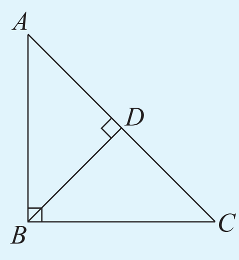
(i) the triangles $ABC$ and $PQC$ are equiangular.

(ii) $\dfrac{QC}{AC} = \dfrac{PQ}{AB} = \dfrac{PC}{BC}$.




**3.** In triangle $ABC$, $\hat{B}$ is a right angle. $BD$ is the perpendicular drawn from $B$ to $AC$. Prove that,

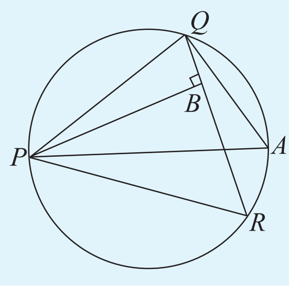
(i) $AB^2 = AD.\,AC$.




**4.** $PA$ is a diameter of the circumcircle of triangle $PQR$. The line $PB$ is the perpendicular drawn from $P$ to $QR$. Prove that

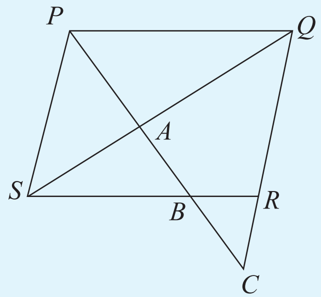
(i) the triangles $PQA$ and $PBR$ are equiangular

(ii) $\dfrac{PQ}{PB} = \dfrac{PA}{PR}$.



5. In parallelogram $PQRS$, the bisector of $Q\hat{P}S$ meets the diagonal $QS$ at $A$, the side $SR$ at $B$ and $QR$ produced at $C$.

Prove that $\frac{PQ}{PS} = \frac{PC}{PB}$.




6. In triangle $ABC$, the point $P$ lies on $AB$ and the point $Q$ lies on $AC$ such that $A\hat{P}Q = A\hat{C}B$. Prove that $AP.AB = AQ.AC$

7. The vertices of a triangle $ABC$ lie on a circle. The bisector of $B\hat{A}C$ intersects the side $BC$ at $Q$ and the circle at $P$. Prove that $AC : AP = AQ : AB$

8. In triangle $ABC$, the bisector of $B\hat{A}C$ meets $BC$ at $D$. The point $X$ lies on $AD$ produced such that $CX = CD$. Prove that,
   (i) the triangles $ACX$ and $ABD$ are equiangular.
   (ii) $\frac{AB}{AC} = \frac{BD}{DC}$

## Miscellaneous Exercise

1. In the rectangle $ABCD$, $E$ is on $DC$ such that $A\hat{E}B = 90°$. Prove that $ADE$, $AEB$ and $EBC$ are similar triangles.

2. In the triangle $ABC$, $\hat{B}$ is a right angle. $AB = 5$ cm and $BC = 2$ cm. The perpendicular bisector of $AC$ intersects the side $AB$ at $Q$. Show that the length of $AQ = 2.9$ cm.

3. In triangle $ABC$, $PQ$ drawn parallel to $BC$ meets the side $AB$ at $P$ and the side $AC$ at $Q$. The lines $CP$ and $BQ$ intersect each other at $S$. $SR$ drawn parallel to $AB$ meets $BC$ at $R$.

Prove that $\frac{BR}{RC} = \frac{AQ}{AC}$.

15

# Data Representation and Interpretation

**By studying this lesson, you will be able to**

- find the limits and boundaries of the intervals of a frequency distribution
- draw the relevant histogram
- draw the relevant frequency polygon
- draw the relevant cumulative frequency curve and find the inter quartile range from that curve

## Limits and boundaries of class intervals

The following is data of the heights (to the nearest centimetre) of 30 students

137, 135, 141, 147, 151, 135, 137, 143, 144, 145
140, 134, 141, 140, 153, 144, 133, 138, 155, 130
136, 137, 142, 143, 145, 143, 154, 146, 148, 158

We know that the range is the value which is obtained by subtracting the least value of the data from the highest value of the data. Therefore,

$$\text{The range of the data} = 158 - 130$$
$$= 28$$

To facilitate interpretation, a group of data is often represented as a frequency distribution. We know that if the range of the data is large, then the data is divided into class intervals. Such a frequency distribution is called a **grouped frequency distrbution**. In such frequency distributions, generally, the number of class intervals should be between 5 and 10. The size of a class interval is determined by dividing the range of the frequency distribution by the number of class intervals and taking the least integer greater than that value .

For example let us group the above data into 6 class intervals. To find the size of a class interval, first divide 28 (the range) by 6 (number of class intervals).

$$\text{then, we obtain } \frac{28}{6} \approx 4.66 .$$

Hence the size of each interval should be 5, which is the least integer greater than 4.66.

Next the first class interval needs to be selected. Since the lowest value of the data is 130, the first class interval should start from 130.

The following are two different grouped frequency distributions from the given data.

| Class intervals | Frequency |
|---|---|
| 130 -135 | 3 |
| 135 - 140 | 7 |
| 140 - 145 | 10 |
| 145 - 150 | 5 |
| 150 - 155 | 3 |
| 155 - 160 | 2 |

First grouped distribution

| Class intervals | Frequency |
|---|---|
| 130 -134 | 3 |
| 135 - 139 | 7 |
| 140 - 144 | 10 |
| 145 - 149 | 5 |
| 150 - 154 | 3 |
| 155 - 159 | 2 |

Second grouped distribution

Consider the first grouped distribution. As an example, the 130 - 135 class interval represents the heights greater than or equal to 130 and less than 135. The second class interval 135 - 140 represents the heights greater than or equal to 135 and less than 140. The other class intervals can be described similarly.

Now consider the second grouped distribution. As an example, the 130 - 134 class interval represents the heights greater than or equal to 130 and less than or equal to 134.

Let us observe another difference between these two distributions. In the first distribution there are no gaps between the class intervals. For example, the class interval 135 - 140 starts from the upper limit 135 of the previous class interval 130 - 135. So there is a common limit for these class intervals. But this is not so in the second distribution. For example 134 is the upper limit of the class interval 130-134, but the next class interval starts from 135. The gap between these two limits is 1.

In the next section of this lesson we hope to learn how to draw histograms. To draw a histogram, there should not be these kinds of gaps. Therefore we must change the second distribution appropriately. This change can be done by introducing a common boundary for the class intervals. That common boundary can be determined easily.

For example, in the second distribution, 134.5 is taken as the boundary of the class intervals 130 - 134 and 135 - 139, which is the exact middle of the upper limit (134) of the class interval 130 - 134 and the lower limit (135) of the class interval 135 - 139.The new distribution constructed in this way is given below.

| Class intervals with boundaries | Frequency |
|---|---|
| 129.5 - 134.5 | 3 |
| 134.5 - 139.5 | 7 |
| 139.5 - 144.5 | 10 |
| 144.5 - 149.5 | 5 |
| 149.5 - 154.5 | 3 |
| 154.5 - 159.5 | 2 |

Here observe that, 0.5 is subtracted from the lower limits and 0.5 is added to the upper limits of every class interval of the original distribution.This rule is valid for the first and last class intervals too. The values 129.5 and 159.5 are obtained in that way. Also observe that, the size of the class interval of this new distribution is 5.

In the above, the first kind of distribution is simple. But practically, the second kind of distribution can be constructed easily. Both kinds of distributions can be found in statistics.

## 15.1 Histogram of a grouped frequency distribution

Let us consider how to draw the histogram of a grouped frequency distribution. A histogram is a graphical representation of a grouped frequency distribution. In a histogram, the frequencies of class intervals are represented by the heights of the rectangular columns which touch each other. Let us consider first, how to draw the histogram if the sizes of the class intervals are equal.(As in the example in the previous section).

When drawing the histogram, follow the steps given below.

- Mark the boundaries of the calss intervals on the horizontal axis drawn to an appropriate scale.
- Along the vertical axis, on an appropriate scale, draw the columns such that the height of the column on a class interval is the corresponding frequency.

By considering the following example, let us observe how to draw the histogram.

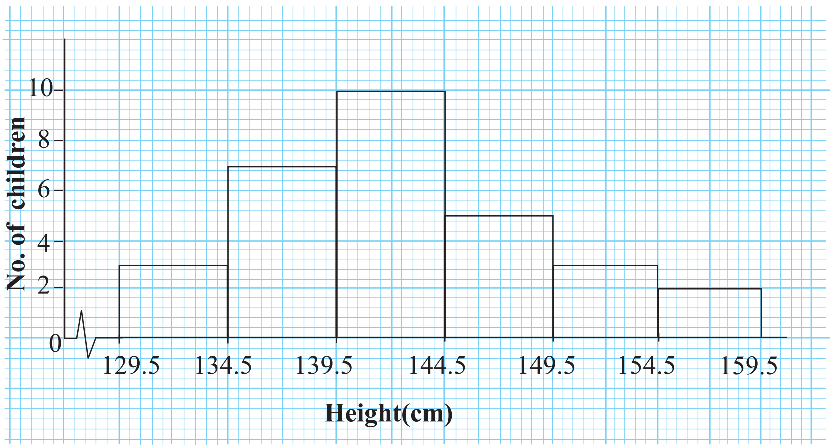
**Example 1**

Draw the histogram of the grouped frequency distribution prepared in the previous section.

For this, let us consider the second frequency distribution.

| Intervals with boundaries | Frequency |
|---|---|
| 129.5 - 134.5 | 3 |
| 134.5 - 139.5 | 7 |
| 139.5 - 144.5 | 10 |
| 144.5 - 149.5 | 5 |
| 149.5 - 154.5 | 3 |
| 154.5 - 159.5 | 2 |

The relevant histogram is given below.
Two small squares along the horizontal axis represents 1 centimetre.
Five small squares along the vertical axis represents two children.




Observe here that the columns touch each other.

Note: Since the data starts from 129.5, it is not necessary to show the class intervals from 0 to 129.5 in the histogram. The mark at the beginning of the $x$ axis indicates that the axis has been shrunk in this region.

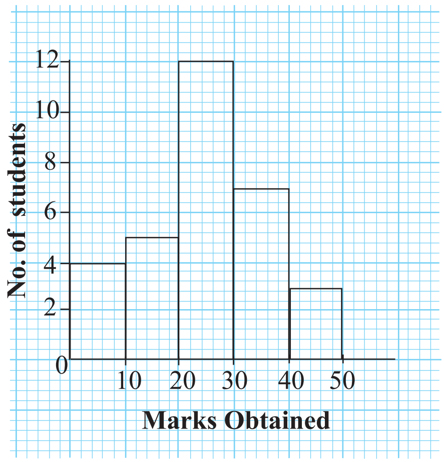
**Example 2**

The following is a frequency distribution of the Mathematics marks of students taken from a School Based Assessment.

| Class intervals (Marks Obtained) | 0 - 10 | 10 - 20 | 20 - 30 | 30 - 40 | 40 - 50 |
|---|---|---|---|---|---|
| Frequency (Number of children) | 4 | 5 | 12 | 7 | 3 |

As an example, the interval 0 - 10 represents the marks greater than or equal to 0 and less than 10. The other class intervals are defined similarly. Draw the histogram of the frequency distribution.

In this frequency distribution, the first class interval ends at 10 and next class interval starts at 10.This histogram can be drawn easily.




Now let us consider how to draw a histogram of a frequency distribution with different sized class intervals.

**Example 3**

A frequency distribution based on the marks of 40 students in a term test is given below.

| Class intervals (Marks Obtained) | 0 - 10 | 10 - 20 | 20 - 30 | 30 - 40 | 40 - 50 | 50 - 70 | 70 - 100 |
|---|---|---|---|---|---|---|---|
| Frequency (Number of students) | 2 | 4 | 6 | 9 | 5 | 8 | 6 |

If you observe the class intervals, you can see that the sizes of the class intervals are not the same. The size of each of the first 5 class intervals is 10 and the sizes of the next two class intervals are 20 and 30 respectively. Another important feature in a histogram is that the areas of the columns are proportional to the relevant frequencies.

Therefore if the sizes of the class intervals are equal, then the frequencies are proportional to the heights of the columns. Hence in the above examples 1 and 2, the frequency can be identified directly with the height of the column. But in this example, since the sizes of the class intervals are not the same, the frequency cannot be identified directly with the height of the column. The heights of the columns should be obtained such that the areas of the columns are proportional to the frequences. This can be done as follows.

In this frequency distribution, the size of all but the class intervals 50 - 70 and 70 - 100 is 10. The size of the class interval 50 - 70 is 20 and the size of the class interval 70 - 100 is 30.

Hence the size of the smallest class interval is 10 and the size of the class interval 50 - 70 is two times that. Since the area of the column which represents the frequency of the class interval should be proportional to the frequency,

$$\text{Height of the column} = \frac{\text{frequency}}{2}$$

$\therefore$ The height the column of the 50 - 70 class interval $= \frac{8}{2}$

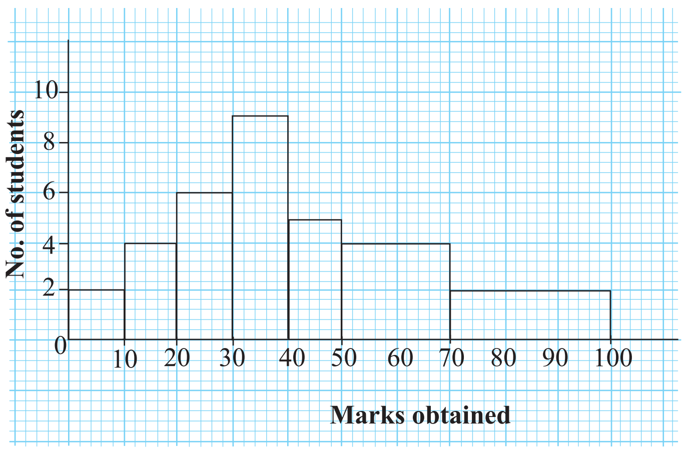
$= 4$

The size of the class interval 70 - 100 is three times the size of the smallest class interval.

$\therefore$ The height the column of the 70 - 100 class interval $= \frac{6}{3}$

$= 2$

After these calculations, the histogram can be drawn as follows.



## Exercise 15.1

**1.** The frequency distribution prepared from the data collected by a weather forecasting center in a certain area is given below. Illustrate this information in a histogram.

| Rainfall in a week in mm. | 10 - 20 | 20 - 30 | 30 - 40 | 40 - 50 | 50 - 60 | 60 - 70 | 70 - 80 |
|---|---|---|---|---|---|---|---|
| Number of weeks | 5 | 6 | 15 | 10 | 7 | 5 | 4 |

**2.** The frequency distribution of the number of books borrowed from a school library in the year 2015 is given below. Illustrate this information in a histogram.

| Class intervals (Number of books issued) | 25 - 29 | 30 - 34 | 35 - 39 | 40 - 44 | 45 - 49 | 50 - 54 |
|---|---|---|---|---|---|---|
| Frequency (Number of days) | 5 | 10 | 20 | 15 | 10 | 7 |

**3.** The frequency distribution prepared from the data obtained by measuring the circumference of teak trees in a forest plantation is given below. Illustrate this information in a histogram.

| Circumference of a tree (cm) | 30 - 34 | 35 - 39 | 40 - 44 | 45 - 49 | 50 - 54 | 55 - 59 |
|---|---|---|---|---|---|---|
| Number of trees | 6 | 8 | 9 | 15 | 24 | 21 |

**4.** The frequency distribution prepared from the data on the daily consumption of water provieded through rural water project to 60 houses is given below. Illustrate this information in a histogram.

| Home usage of water (Litres) | 8 - 12 | 13 - 17 | 18 - 22 | 23 - 27 | 28 - 32 | 33 - 37 | 38 - 42 |
|---|---|---|---|---|---|---|---|
| Number of houses | 4 | 6 | 15 | 15 | 10 | 7 | 3 |

**5.** The information on the monthly electricity consumption of 75 houses in January 2015 is shown in the table given below. Illustrate this information in a histogram.

| Class intervals (Units of electricity) | 10 - 20 | 20 - 30 | 30 - 40 | 40 - 50 | 50 - 60 | 60 - 100 |
|---|---|---|---|---|---|---|
| Frequency (Number of houses) | 10 | 11 | 14 | 16 | 12 | 12 |

**6.** The following frequency distribution shows the information on the number of telephone calls and the time duration of each telephone call on a certain day at a communication centre.

| Time duration of one telephone call (seconds) | 30 - 45 | 45 - 60 | 60 - 75 | 75 - 90 | 90 - 120 |
|---|---|---|---|---|---|
| Number of calls | 8 | 9 | 12 | 16 | 8 |

## 15.2 Frequency polygon

A frequency polygon is a graphical representation of grouped data similar to a histogram. There are two methods to construct frequency polygons.

- From the histogram of the frequency distribution
- From the mid-values and frequencies of the class intervals

In the following example, let us consider first, how to construct a frequency polygon from the histogram.

**Example 1**

Let us consider a frequency distribution used in a previous example.

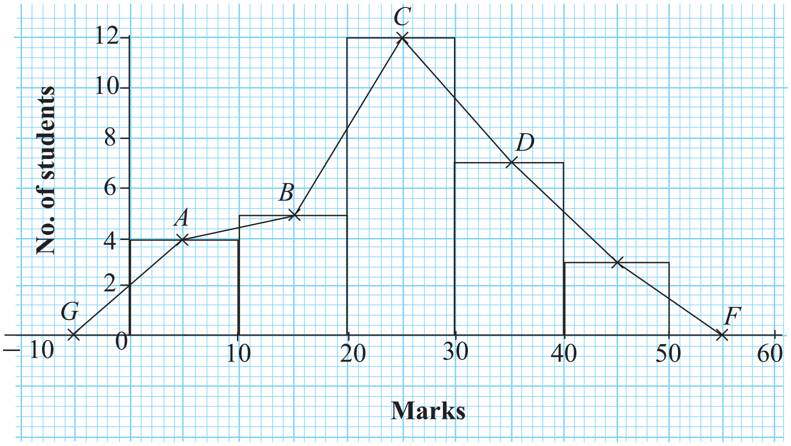
| Marks | 0 - 10 | 10 - 20 | 20 - 30 | 30 - 40 | 40 - 50 |
|---|---|---|---|---|---|
| Number of students | 4 | 5 | 12 | 7 | 3 |

(i) First draw the histogram relevant to the given information

(ii) In each column in the histogram, mark the sign "×" at the middle of the top of the column. (See the following figure; the signs "×" denoted as $A$, $B$, $C$, $D$ and $E$ )

(iii) Join these "×"signs by the line segments as shown in the figure.

(iv) On the horizontal axis, mark half the length of the class interval to the right of the last class interval and to the left of the first class interval (hence 5 units). Join *E F* and *A G*.




Now you get the polygon *ABCDEFG*. This polygon is called the frequency polygon of the frequency distribution. If you observe carefully, you can see that the area of the frequency polygon is equal to the sum of the areas of the columns of the histogram.

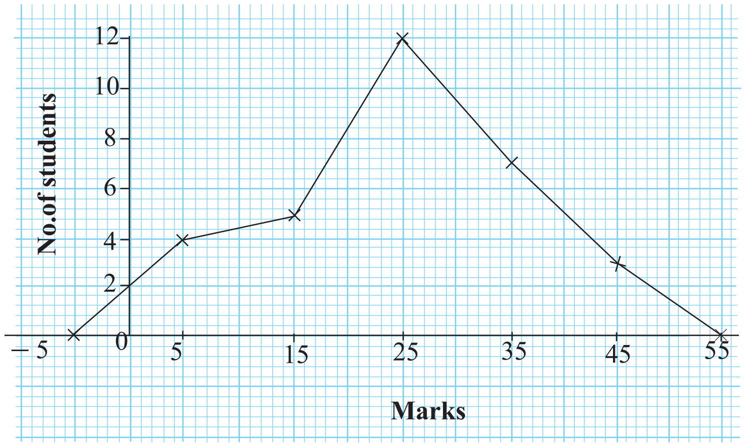
To draw a frequency polygon, it is not necessary to draw the histogram first. The frequency polygon can be drawn directly by using the mid-values of the class intervals and using the relevant frequencies. The following example shows how to draw the frequency polygon using this method.

**Example 2**

From the given frequency distribution, prepare a table with the mid-values of the class intervals to draw the frequency polygon.

| Class interval | Mid-value | Frequency |
|---|---|---|
| 0 - 10 | 5 | 4 |
| 10 - 20 | 15 | 5 |
| 20 - 30 | 25 | 12 |
| 30 - 40 | 35 | 7 |
| 40 - 50 | 45 | 3 |

Mark the mid-values of the class intervals along the horizontal axis and mark the frequencies along the vertical axis. Then mark the corresponding points. The frequency polygon can be obtained by joining those points respectively by line segments.



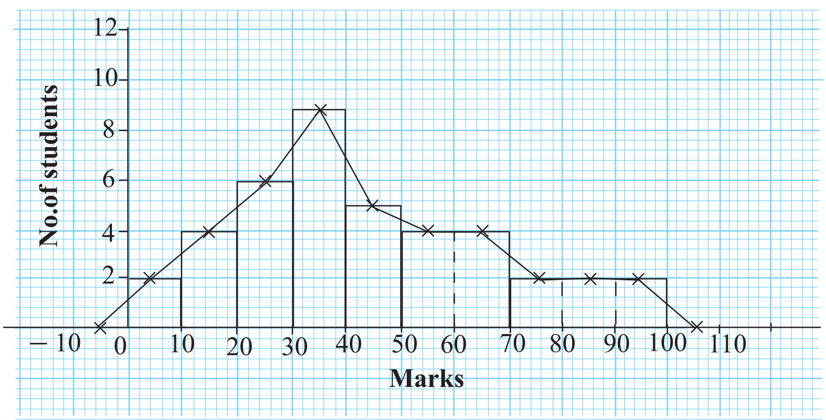

Let us consider next, how to draw a frequency polygon with unequal sized class intervals.

**Example 3**

| Class intervals (Obtained marks) | 0 - 10 | 10 - 20 | 20 - 30 | 30 - 40 | 40 - 50 | 50 - 70 | 70 - 100 |
|---|---|---|---|---|---|---|---|
| Frequency (Number of students) | 2 | 4 | 6 | 9 | 5 | 8 | 6 |

Let us draw the frequency polygon for the above frequency distribution with unequal sized class intervals.

The relevant frequency polygon is given below.




Here, the class interval 50 - 60 of size 20 is divided in to two class intervals of size 10 and as the relevant frequency, the frequency of the mid-point of each class interval is considered. Similarly the class interval 70 - 100 of size 30 is divided in to three class intervals of size 10 and as the relevant frequency, the frequency of the mid-point of each class interval is considered. Also in this case, observe that the area of the frequency polygon is equal to the sum of the areas of columns of the histogram.

**Exercise 15.2**

**1.** The frequency distribution prepared from the data collected by measuring the masses of students who attended a medical clinic of a school is given below.

| Mass of a student (kg) | 30 - 35 | 35 - 40 | 40 - 45 | 45 - 50 | 50 - 55 |
|---|---|---|---|---|---|
| Number of students | 8 | 10 | 15 | 7 | 15 |

(i) Illustrate this information in a histogram.
(ii) Draw the frequency polygon on this histogram.

**2.** The following is a frequency distribution prepared from the data collected through a test conducted to determine the life time of bulbs manufactured by a company.

| Class intervals (No. of hours the bulb lighted) | 100 - 300 | 300 - 400 | 400 - 500 | 500 - 600 | 600 - 700 | 700 - 800 |
|---|---|---|---|---|---|---|
| Frequency (Number of bulbs) | 12 | 10 | 20 | 25 | 15 | 12 |

(i) Illustrate this information in a histogram.
(ii) Draw the frequency polygon on this histogram.

**3.** Information on the mass of the members of a sport club is given in the following table.

| Mass (kg) | 60 - 65 | 65 - 70 | 70 - 75 | 75 - 80 | 80 - 85 |
|---|---|---|---|---|---|
| Number of members | 10 | 15 | 6 | 4 | 2 |

(i) From this information prepare a table with the mid-values of the class intervals.
(ii) Draw the frequency polygon using the mid-values of the class intervals.

**4.** The following is a frequency distribution prepared from the Mathematics marks of grade11 students in a school.

| Class intervals (Marks) | 0 - 30 | 30 - 40 | 40 - 50 | 50 - 60 | 60 - 100 |
|---|---|---|---|---|---|
| Frequency (Number of students) | 6 | 5 | 10 | 7 | 12 |

(i) Draw the histogram of this information and use it to draw the frequency polygon.

**5.** The table given below is prepared from the information of a communication centre on the number of telephone calls taken on a certain day and the time duration of those calls.

| Time duration of one telephone call (seconds) | 1 - 4 | 4 - 7 | 7 - 10 | 10 - 13 | 13 - 16 |
|---|---|---|---|---|---|
| Number of calls | 3 | 9 | 20 | 12 | 6 |

(i) Draw the histogram of this frequency distribution.
(ii) Use this histogram to draw the frequency polygon.

## 15.3 Cumulative frequency curve of a grouped frequency distribution

This is a another method of representing data of a frequency distribution graphically.

Let us see how to draw the cumulative frequency curve by considering the following example

**Example 1**

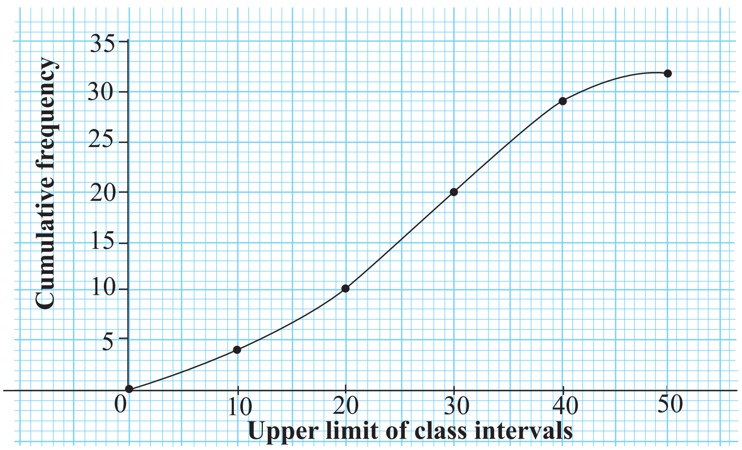
The following is a frequency distribution of the marks in Mathematics of 32 students in a class. Let us draw its cumulative frequency curve.

| Marks | 0 - 10 | 10 - 20 | 20 - 30 | 30 - 40 | 40 - 50 |
|---|---|---|---|---|---|
| Number of students | 4 | 6 | 10 | 9 | 3 |

Let us construct a cumulative frequency table from the above table.

| Class intervals | Frequency | Cumulative frequency |
|---|---|---|
| 0 - 10 | 4 | 4 |
| 10 - 20 | 6 | 10 |
| 20 - 30 | 10 | 20 |
| 30 - 40 | 9 | 29 |
| 40 - 50 | 3 | 32 |

The word "cumulative" means "increasing by addition". For example, in the above table, the cumulative frequency relevant to the class interval 20-30 is the sum of all frequencies less than in the intervals up to the class mark 30. (In other words no. of students whose marks are less than 30) That is equal to 20. The cumulative frequency relevant to the class interval 40 - 50 is the number of students whose marks are less than 50. That is 32, the total number of students. After completing this table, to draw the cumulative frequency curve, all the points corresponding to the upper limit of each class interval via the relevant cumulative frequency should be marked. These points should be joined smoothly, as shown below.




## 15.4 Quartiles and interquartile range of a frequency distribution

In previous sections, we learnt how to construct a histogram, frequency polygon and cumulative frequency curve from a group of data. These are useful to get an idea about how far the data disperses centrally. For example, just by looking at the histogram, the modal class of a grouped frequency distribution can be decided. Similarly, an idea on how far the data disperses symmetrically can also be obtained. In this section, we hope to learn about quartiles and the interquartile range of a group of data. From this we can get some idea about the dispersion of data.

The first thing to do to find quartiles and the interquartile range is to organize the data in ascending order. After that the first quartile ($Q_1$), the second quartile ($Q_2$) and the third quartile ($Q_3$) can be found as follows.

Step **1**: First find the median of the data. This is the second quartile.

Step **2**: Find the median of the data to the left of the second quartile. This is the first quartile.

Step **3**: Find the median of the data to the right of the second quartile. This is the third quartile.

As an example, consider the following group of data which is in ascending order.

**Example 1**

5, 6, 6, 8, 11, 12, 12, 12, 13, 14, 14, 14, 17, 18, 20, 24, 25, 26, 30

Here,there are 19 data. The median of these is 14 (It is in the square)

5, 6, 6, 8, 11, 12, 12, 12, 13, [14,] 14, 14, 17, 18, 20, 24, 25, 26, 30

Now consider the data left of the median.

5, 6, 6, 8, [11,] 12, 12, 12, 13

The median of that part is 11. That is also in the square.
At last, consider the data right of the median. .

14, 14, 17, 18, [20,] 24, 25, 26, 30

The median of that part is 20. That is also in a square.
Hence,

first quartile $= Q_1 = 11$

second quartile $= Q_2 = 14$

third quartile $= Q_3 = 20$.

**Example 2**

Let us find the quartiles of the following 18 data written in ascending order .

2, 2, 3, 6, 6, 6, 7, 8, 8, 11, 11, 12, 12, 15, 15, 16, 17, 20

2, 2, 3, 6, 6, 6, 7, 8, [8, 11,] 11, 12, 12, 15, 15, 16, 17, 20

Its median is the mean of the 8 and 11 in the rectangle.

Hence,

$$Q_2 = \frac{8 + 11}{2} = 9.5$$

The left side of the median is:

2, 2, 3, 6, [6,] 6, 7, 8, 8

The median of that is 6. It is in the square.
Hence $Q_1 = 6$.

At last, the right side of the median is:

11, 11, 12, 12, [15,] 15, 16, 17, 20

The median of that is 15. It is in the square.
Hence $Q_3 = 15$.

**Example 3**

There are 17 data in the following data string. Find the quartiles.

102, 104, 104, 105, 107, 107, 107, 108, 112, 112, 113, 115, 115, 119, 120, 125, 126

The heads of the arrows indicate the quartiles which are calculate according to the above steps.

102, 104, 104, [105, 107,] 107, 107, 108, [112,] 112, 113, 115, [115, 119,] 120, 125, 126

↑ ↑ ↑

$$Q_1 = \frac{105 + 107}{2} = 106$$

$$Q_2 = 112$$

$$Q_3 = \frac{115 + 119}{2} = 117$$

**Example 4**

There are 16 data in the following data string. Observe how to calculate the quartiles which are indicated by the heads of arrows.

21, 23, 25, [25, 26,] 28, 28, [30, 30,] 34, 34, [35, 37,] 37, 40, 42
↑ ↑ ↑

Hence, $Q_1 = \frac{25 + 26}{2} = 25.5$, $Q_2 = \frac{30 + 30}{2} = 30$, $Q_3 = \frac{35 + 37}{2} = 36$.

In statistics, there are several methods to find quartiles of a data string. The method described here is the most practical and the most convenient method.

Another method to calculate quartiles is, finding the locations of the quartiles using the formulae

$Q_1$ in the $\frac{1}{4}$(n + 1) position, $Q_2$ in the $\frac{2}{4}$(n + 1) position, $Q_3$ in the $\frac{3}{4}$(n + 1) position.

For example, consider the data string 4 6 7 8 15 18 20.

In this data string, according to the formulae,

$Q_1$ is located at the place $\frac{1}{4}(7 + 1) = 2$. Hence $Q_1 = 6$.

$Q_2$ is located at the place $\frac{2}{4}(7 + 1) = 4$. Hence $Q_2 = 8$.

$Q_3$ is located at the place $\frac{3}{4}(7 + 1) = 6$. Hence $Q_3 = 18$.

As another example, consider the data string 9 12 18 20 21 23 24 26.
In this data string, according to the formulae,

$Q_1$ is located at the place $\frac{1}{4}(8+1) = 2.25$. Hence $Q_1 = 12 + \frac{1}{4}(18-12) = 13.5$

$Q_2$ is located at the place $\frac{2}{4}(8+1) = 4.5$. Hence $Q_2 = \frac{20+21}{2} = 20.5$

$Q_3$ is located at the place $\frac{3}{4}(8+1) = 6.75$. Hence $Q_3 = 23 + \frac{3}{4}(24-23) = 23.75$

Here, when using different methods from each other, different answers with small deviations may be obtained. But this is not a problem since in methods of statistics it is expected to get approximate values.

The interquartile range of a group of data is the value obtained by subtracting the first quartile from the third quartile.

Hence **Interquartile range = $Q_3 - Q_1$**

**Exercise 15.4**

**1.** The following are ages of 17 workers in a work place prepared in ascending order.

21, 22, 23, 24, 25, 27, 27, 30, 34, 35, 40, 41, 42, 44, 46, 47, 50

For this group of data, find the following.

(i) Median
(ii) First quartile
(iii) Third quartile
(iv) Interquartile range.

**2.** The number of members in the families of the students in a class is given below.

7, 6, 4, 3, 8, 5, 5, 4, 3, 6, 4, 6, 7, 10, 5

Prepare this group of data in ascending order and find the following.

(i) Median
(ii) First quartile
(iii) Third quartile
(iv) Interquartile range.

**3.** The information on the electricity consumption of 32 shops in a town during a day in the year 2015 of 32 shops in a town is given below.

| No. of units | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 10 |
|---|---|---|---|---|---|---|---|---|
| No. of shops | 5 | 2 | 6 | 6 | 7 | 2 | 3 | 1 |

For this group of data, find the following

(i) Median

(ii) First quartile

(iii) Third quartile

(iv) Interquartile range.

## More on Interquartile Range

In this section, we hope to learn how to find quartiles and the interquartile range of grouped data. Here we describe only, how to find it using the cumulative frequency curve. Let us consider how to find quartiles and the interquartile range of grouped data from the following example.

**Example 1**

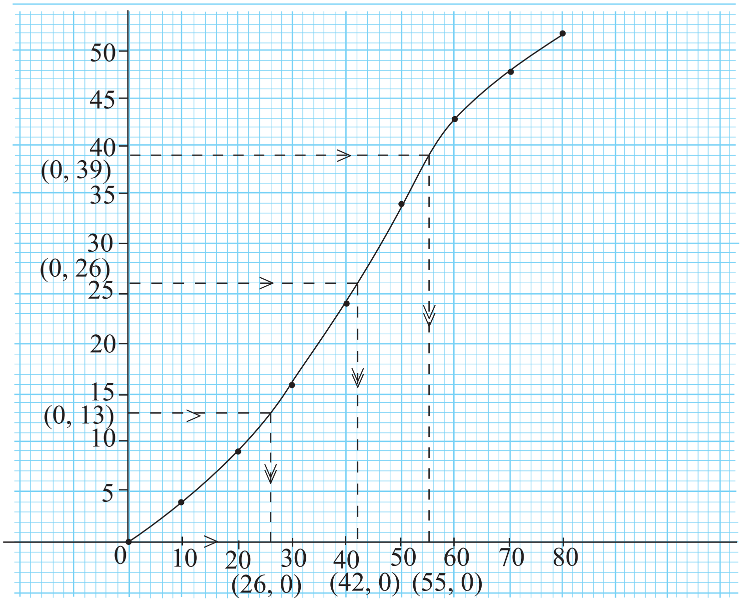
The following is a frequency distribution prepared from the Mathematics marks of a group of students in grade 11. Let us draw the cumulative frequency curve for this frequency distribution.

| Marks | 0 - 10 | 10 - 20 | 20 - 30 | 30 - 40 | 40 - 50 | 50 - 60 | 60 - 70 | 70 - 80 |
|---|---|---|---|---|---|---|---|---|
| No. of students | 4 | 5 | 7 | 8 | 10 | 9 | 5 | 4 |

Let us construct a table from this data to draw the cumulative frequency curve.

| Class intervals | Frequency | Cumulative frequency |
|---|---|---|
| 0 - 10 | 4 | 4 |
| 10 - 20 | 5 | 9 |
| 20 - 30 | 7 | 16 |
| 30 - 40 | 8 | 24 |
| 40 - 50 | 10 | 34 |
| 50 - 60 | 9 | 43 |
| 60 - 70 | 5 | 48 |
| 70 - 80 | 4 | 52 |

Let us draw the cumulative frequency curve as learnt in section 15.3.




Now let us pay attention to the vertical lines and the horizontal lines in the above figure of the cumulative frequency curve.
Here the total number of data is 52. That is, the sum of the frequencies is 52. Let us first find, the locations of the first, second and third quartiles of those 52 data.

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Note: When finding the quartiles from the cumulative frequency curve, there is no need to do it as in setion 15.5. Since the number of data is large (More than 30 data values is considered as a large number of data), it is sufficient to locate the positions of $\frac{1}{4}$ of the frequencies, $\frac{1}{2}$ of the frequencies and $\frac{3}{4}$ of the frequencies.

---

When the cumulative frequency is increasing then the first quartile is located at the position of $\frac{1}{4}$ th of the total frequency. Hence,

$$\text{the position of } Q_1 = \frac{1}{4} \times 52^{\text{th}} \text{ position} = 13^{\text{th}} \text{ position}$$

$$\text{the position of } Q_2 = \frac{1}{2} \times 52^{\text{th}} \text{ position} = 26^{\text{th}} \text{ position}$$

$$\text{the position of } Q_3 = \frac{3}{4} \times 52^{\text{th}} \text{ position} = 39^{\text{th}} \text{ position}$$

Now it is needed to find the data corresponding to the points 13, 26 and 39 (frequencies) of the vertical axis. The necessary lines are illustrated in the figure. As an example the first quartile can be found as follows.

Since the first quartile is located at the 13th position, a horizontal line is drawn from the point 13 on the vertical axis until it meets the curve. From the point that line meets the curve, a vertical line is drawn until it meets the horizontal axis. The value of that point on the horizontal axis is the first quartile.

If we find the quartiles for the given example, then we get $Q_1 = 26$, $Q_2 = 42$ and $Q_3 = 55$.

Hence, the interquartile range $= Q_3 - Q_1 = 55 - 26 = 29$.

As an example, if the total frequency of a frequency distribution is 51 then the first, the second and the third quartiles are in the

$\frac{1}{4} \times 51 = 12.75^{th}$ position

$\frac{1}{2} \times 51 = 25.5^{th}$ position

$\frac{3}{4} \times 51 = 38.25^{th}$ position respectively.

Hence the quartiles can be found by considering the values on the $x$-axis corresponding to the values 12.75, 25.5 and 38.25 (or the appropriate values rounded to the scale of the graph) on the vertical axis.

## Exercise 15.5

**1.** The following is the information on the leave taken by the employees of a certain office in the year 2015.

| Number of days | 0 - 4 | 4 - 8 | 8 - 12 | 12 - 16 | 16 - 20 | 20 - 24 |
|---|---|---|---|---|---|---|
| Number of employees | 10 | 18 | 11 | 8 | 5 | 4 |

(i) Construct the cumulative frequency table with the above information.
(ii) Draw the cumulative frequency curve from the table.
(iii) From the cumulative frequency curve find the following.
(*a*) Median of the leave taken by the employees.
(*b*) Interquartile range of the data.

**2.** The following is a table of marks obtained in a monthly test of grade 11 students for science.

| Class interval of marks | 0 - 15 | 15 - 30 | 30 - 45 | 45 - 60 | 60 - 75 | 75 - 90 |
|---|---|---|---|---|---|---|
| No. of students | 6 | 8 | 12 | 20 | 10 | 4 |

(i) From the data of the table, construct a cumulative frequency table.
(ii) Draw the cumulative frequency curve.
(iii) From the cumulative frequency curve find the following.
($a$) First quartile
($b$) Second quartile
($c$) Third quartile.
(iv) Find the interquartile range of the marks.

**3.** The following is information on the salaries of employees of a garment factory, in the month of January 2015. Using this information, draw the cumulative frequency curve. From that curve, find the median of salary of an employee and find the interquartile range of the salaries.

| Monthly salary of an employee (Rupees) (Class interval) | 20000 - 20500 | 20500 - 21000 | 21000 - 21500 | 21500 - 22000 | 22000 - 22500 | 22500 - 23000 | 23000 - 23500 | 23500 - 24000 |
|---|---|---|---|---|---|---|---|---|
| No. of employees | 8 | 10 | 15 | 18 | 25 | 12 | 9 | 7 |

## Miscellaneous Exercise

**1.** The following table is prepared based on the monthly charges for the consumption of electricity of houses in a housing scheme.

| Monthly charges (Rupees) | 0 - 200 | 200 - 400 | 400 - 600 | 600 - 800 | 800 - 1000 |
|---|---|---|---|---|---|
| No. of houses | 8 | 14 | 24 | 12 | 6 |

(i) From this information, construct a cumulative frequency table.
(ii) Draw the cumulative frequency curve.
(iii) Find the median.
(iv) Find the interquartile range.

**2.** The following frequency distribution is prepared from the information on the ages of the staff of a certain office.

| Age (years) | 20 - 25 | 25 - 30 | 30 - 35 | 35 - 40 | 40 - 45 | 45 - 50 | 50 - 55 | 55 - 60 |
|---|---|---|---|---|---|---|---|---|
| Number of employees | 8 | 12 | 14 | 18 | 16 | 6 | 2 | 2 |

For the given grouped frequency distribution,

(i) draw the histogram

(ii) draw the frequency polygon.

(iii) draw the cumulative frequency curve.

(iv) Find the interquartile range from the cumulative frequency curve.

**3.** The following table is prepared from information on the water consumption of 100 houses in a housing scheme during a certain month.

| No of units | 20 - 29 | 30 - 39 | 40 - 49 | 50 - 59 | 60 - 69 | 70 - 79 |
|---|---|---|---|---|---|---|
| No of houses | 2 | 8 | 35 | 40 | 10 | 5 |

(i) From this information, draw the histogram and the frequency polygon.

(ii) Construct a cumulative frequency table.

(iii) From that table, draw the cumulative frequency curve.

(iv) Find the interquartile range of this information.

# 16 Geometric progressions

**By studying this lesson you will be able to,**

- identify whether a given number sequence is a geometric progression,
- use the formula for the $n^{\text{th}}$ term of a geometric progression,
- use the formula for the sum of the first $n$ terms of a geometric progression and
- solve problems related to geometric progressions.

## 16.1 Geometric Progressions

Let us first recall what we learnt in grade 10 about arithmetic progressions. Given below is an arithmetic progression.

5, 7, 9, 11, ...

In this sequence, by adding the constant value 2 to any term, the term next to it is obtained. We called this constant value the common difference of the arithmetic progression.

Now, careful observe the sequence given below.

3, 6, 12, 24, 48, 96, ...

In this sequence, the first term is 3. The second term is obtained by multiplying the first term by 2, third term is obtained by multiplying the second term by 2. The sequence continues in this manner, where, each term multiplied by 2 is the next term. In other words, if we divide any term other than the first term, by the previous term we get the constant value 2.

A sequence that yields a fixed value when any term, other than the first term, is divided by the previous term is called a **geometric progression**. The fixed value, by which each term is multiplied to obtain the next term, is called the **common ratio** of the geometric progression. The common ratio of the above geometric progression is 2.

Given a number sequence, one can check whether it is a geometric progression by doing the following test. Note down the value obtained by dividing the second term by the first term. Similarly, note down the values obtained by dividing the third term by the second term, fourth term by the third term and so forth. If the values noted down are all equal, then it is a geometric progression. When the noted values are all equal, this common value is the **common ratio**.

**Example 1**

Check whether the number sequence 2, 6, 18, 54, ... is a geometric progression.

$$\frac{6}{2} = 3, \qquad \frac{18}{6} = 3, \qquad \frac{54}{18} = 3$$

$$\therefore \quad \frac{6}{2} = \frac{18}{6} = \frac{54}{18} = 3$$

$\therefore$ The given sequence is a geometric progression. Moreover, the common ratio is 3.

**Example 2**

Check whether the number sequence 200, 100, 50, 20, ... is a geometric progression.

$$\frac{100}{200} = \frac{1}{2}, \qquad \frac{50}{100} = \frac{1}{2}, \qquad \frac{20}{50} = \frac{2}{5}$$

Because the ratios are not equal, this sequence is not a geometric progression.

**Example 3**

Check whether the number sequence 5, – 10, 20, – 40, 80, ... is a geometric progression.

$$\frac{-10}{5} = -2, \qquad \frac{20}{-10} = -2, \qquad \frac{-40}{20} = -2, \qquad \frac{80}{-40} = -2$$

$$\therefore \quad \frac{-10}{5} = \frac{20}{-10} = \frac{-40}{20} = \frac{80}{-40} = -2$$

$\therefore$ This is a geometric progression with common ratio – 2.

**Example 4**

The terms 4, $x$, 16 are consecutive terms of a geometric progression. What is the value of $x$?

$\frac{x}{4} = \frac{16}{x}$ because the terms are in a geometric progression. We can find the value of $x$ by solving this equation.

If $\frac{x}{4} = \frac{16}{x}$, then $x^2 = 64$.

Thus $x^2 - 8^2 = 0$

$\therefore$ $(x - 8)(x + 8) = 0$

$\therefore$ $x = 8$ or $x = -8$

Let us check for each of the values we have obtained, whether the three terms 4, $x$, 16 are in a geometric progression.

When $x = 8$, the sequence 4, 8, 16 is a geometric progression with common ratio 2. When $x = -8$, the sequence 4, $-8$, 16 is a geometric progression with a common ratio $-2$.

**Exercise 16.1**

**1.** Select and write down the geometric progressions from the number sequences given below.

**(a)** 2, 4, 8, ... **(b)** $-6, -18, -54, \ldots$ **(c)** 64, 32, 16, 8, ...

**(d)** 5, 10, 30, 120, ... **(e)** $-2, 6, -18, 54, \ldots$ **(f)** $81, 27, 3, \frac{1}{9}, \ldots$

**(g)** 0.0002, 0.002, 0.02, 0.2, ... **(h)** $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{36}, \frac{1}{72}, \ldots$

## 16.2 $n^{th}$ term of a Geometric Progression

Recall that you learnt, in Grade 10, that the $n^{th}$ term of an arithmetic progression with first term $a$ and common difference $d$ is $T_n = a + (n-1)d$.

Let us now consider how we can obtain an expression for the $n^{th}$ term of a geometric progression. We will write "$a$" for the first term of the geometric progression and "$r$" for the common ratio. Moreover, we will denote the $n^{th}$ term of the sequence by $T_n$.

Consider the geometric progression 2, 6, 18, 54, ... . Here, the first term $(a)$ is 2 and the common ratio $(r)$ is 3.

Carefully observe that;

$$T_1 = 2 = 2 \times 1 = 2 \times 3^{1-1}$$
$$T_2 = 6 = 2 \times 3 = 2 \times 3^{2-1}$$
$$T_3 = 18 = 2 \times 3 \times 3 = 2 \times 3^{3-1}$$
$$T_4 = 54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^{4-1}$$

The above expressions can be written in terms of the first term $(a)$ and the common ratio $(r)$, as

$$T_1 = 2 \times 3^0 = a \times r^{1-1}$$
$$T_2 = 2 \times 3^1 = a \times r^{2-1}$$
$$T_3 = 2 \times 3^2 = a \times r^{3-1}$$
$$T_4 = 2 \times 3^3 = a \times r^{4-1}$$

According to the observable pattern, the $n^{\text{th}}$ term $T_n$ can be written as $T_n = ar^{n-1}$.

> The $n^{\text{th}}$ term of the geometric progression with first term ***a*** and common ratio ***r*** is given by
> $$\boldsymbol{T_n = ar^{n-1}}.$$

**Example 1**

Find the 5th term of the geometric progression with first term 3 and common ratio 2.

$a = 3,\ r = 2,\ n = 5$

$$\begin{aligned} T_n &= ar^{n-1} \\ T_5 &= 3 \times 2^{5-1} \\ &= 3 \times 2^4 \\ &= 3 \times 16 \\ &= 48 \end{aligned}$$

Therefore, the 5th term is 48.

**Example 2**

Find the fifth and the seventh terms of the geometric progression 81, 27, 9,...

$a = 81$

$r = \dfrac{27}{81} = \dfrac{1}{3}$

$T_n = ar^{n-1}$

$$\therefore \quad \begin{aligned} T_5 &= 81 \times \left(\frac{1}{3}\right)^{5-1} \\ &= 81 \times \left(\frac{1}{3}\right)^4 \\ &= 81 \times \frac{1}{81} \\ &= 1 \end{aligned}$$

$$\begin{aligned} T_7 &= 81 \times \left(\frac{1}{3}\right)^{7-1} \\ &= 81 \times \left(\frac{1}{3}\right)^6 \\ &= 81 \times \frac{1}{729} \\ &= \frac{1}{9} \end{aligned}$$

Thus, the fifth term is 1 and the seventh term is $\dfrac{1}{9}$.

**Exercise 16.2**

1. Find the $6^{th}$ term of the geometric progression with first term 5 and common ratio 2.
2. Find the $6^{th}$ and $8^{th}$ terms of the geometric progression with first term 4 and common ratio – 2.
3. Find the $4^{th}$ and $7^{th}$ terms of the geometric progression with first term – 2 and common ratio – 3.
4. Of a geometric progression, the first term is 1000 and the common ratio is $\frac{1}{5}$. Find the $6^{th}$ term.
5. Find the $6^{th}$ term of the geometric progression 0.0002, 0.002, 0.02,... .
6. Find the $5^{th}$ term of the geometric progression $\frac{3}{8}, \frac{3}{4}, 1\frac{1}{2}, \ldots$ .
7. Find the $4^{th}$ term of the geometric progression 75, – 30, 12,... .
8. Find the $7^{th}$ term of the geometric progression 192, 96, 48,... .
9. Find the $9^{th}$ term of the geometric progression 0.6, 0.3, 0.15,...
10. Find the $10^{th}$ term of the geometric progression 8, 12, 18,...

## 16.3 Using the formula $T_n = ar^{n-1}$

When all but one from the first term ($a$), common ratio ($r$), $n^{th}$ term $T_n$ and $n$, of a geometric progression are given, the unknown value can be found by substituting the given values into $T_n = ar^{n-1}$.

consider at the following examples.

**Example 1**

Find the first term of the geometric progression with common ratio 3 and $4^{th}$ term 54.

$r = 3,\ n = 4,\ T_n = 54$

$$T_n = ar^{n-1}$$
$$\therefore\ T_4 = a \times (3)^{4-1}$$
$$\therefore\ 54 = a \times (3)^3$$
$$\therefore\ 54 = a \times 27$$
$$\therefore\ a = \frac{54}{27}$$
$$= 2$$

The first term of the sequence is 2.

**Example 2**

Find the common ratio of the geometric progression with first term 5 and 7th term 320, and list the first five terms.

$a = 5,\ n = 7,\ T_7 = 320$

$$T_n = ar^{(n-1)}$$
$$T_7 = 5 \times (r)^{7-1}$$
$$\therefore\ 320 = 5 \times (r)^6$$
$$\therefore\ r^6 = \frac{320}{5}$$
$$= 64$$
$$= (+2)^6 \text{ or } (-2)^6$$
$$\therefore\ r = 2 \text{ or } -2$$

There are two values for the common ratio. Hence, there are two geometric progressions satisfying the given conditions.

First five terms of the progression with $r = 2$ are 5, 10, 20, 40, 80.
First five terms of the progression with $r = -2$ are 5, – 10, 20, – 40, 80.

**Example 3**

Which term is $\frac{1}{64}$, of the geometric progression with first term 64 and common ratio $\frac{1}{4}$?

$a = 64,\ r = \frac{1}{4},\ T_n = \frac{1}{64}$

$$T_n = ar^{n-1}$$
$$\frac{1}{64} = 64 \times \left(\frac{1}{4}\right)^{(n-1)}$$
$$\left(\frac{1}{4}\right)^{(n-1)} = \frac{1}{64 \times 64}$$
$$\left(\frac{1}{4}\right)^{(n-1)} = \frac{1}{4^6}$$
$$\left(\frac{1}{4}\right)^{(n-1)} = \left(\frac{1}{4}\right)^6$$
$$(n-1) = 6$$
$$n = 6 + 1$$
$$= 7 \quad \therefore \text{ it is the 7th term that is equal to } \frac{1}{64}.$$

**Example 4**

The first term of a geometric progression, is 160 and the 6th term is 1215. Find the common ratio of the progression.

$a = 160, \ T_6 = 1215, n = 6$

$$T_n = ar^{(n-1)}$$

$$1215 = 160\,(r)^{6-1}$$
$$160r^5 = 1215$$
$$\therefore \quad r^5 = \frac{1215}{160}$$
$$= \frac{243}{32}$$
$$= \frac{3^5}{2^5}$$
$$= \left(\frac{3}{2}\right)^5$$
$$\therefore \ r = \frac{3}{2}$$
$$= 1\frac{1}{2}$$

$\therefore$ The common ratio is $1\frac{1}{2}$.

Similarly, when any two terms of a geometric progression are given, $T_n = ar^{n-1}$ can be used to find the first term and the common ratio. Consider the following example.

**Example 5**

Find the common ratio and the first term of the geometric progression with 3rd term 48 and 6th term 3072.

Let us first construct two equations from the given information.

$$T_n = ar^{n-1}$$
$$T_3 = ar^{(3-1)}$$
$$ar^2 = 48 \text{ ——— (1)}$$
$$T_6 = ar^{(6-1)}$$
$$ar^5 = 3072 \text{ ——— (2)}$$

Both unknowns, $a$ and $r$, appear in both equations 1 and 2. We can easily remove $a$ from these two by dividing the equations.

② ÷ ①
$$\frac{ar^5}{ar^2} = \frac{3072}{48}$$
$$r^3 = 64$$
$$r^3 = 4^3$$
$$r = 4$$

Substitute $r = 4$ to ①
$$ar^2 = 48$$
$$a(4)^2 = 48$$
$$16a = 48$$
$$a = \frac{48}{16}$$
$$a = 3$$

First term of the progression = 3

Common ratio = 4

**Example 6**

The 6th term of a geometric progression is – 8 and the 10th term of the same progression is – 128.

(i) Show that there are two such geometric progressions.

(ii) Write down the first five term of each progression.

(i) $T_n = ar^{(n-1)}$
$$T_6 = ar^{(6-1)}$$
$$ar^5 = -8 \text{ ——— ①}$$
$$T_{10} = ar^{(10-1)}$$
$$ar^9 = -128 \text{ ——— ②}$$

② ÷ ①
$$\frac{ar^9}{ar^5} = \frac{-128}{-8}$$
$$r^4 = 16$$
$$r^4 = 2^4 \text{ or } (-2)^4$$
$$r = 2 \text{ or } -2$$

Because there are two values for the common ratio, there are two such progressions.

(ii) Substituting $r = 2$, to ①
$$ar^5 = -8$$
$$a(2)^5 = -8$$
$$a \times 32 = -8$$

$$a = \frac{-8}{32}$$

$$a = -\frac{1}{4}$$

First five terms of the geometric progression with $r = 2$ and $a = -\frac{1}{4}$ are $-\frac{1}{4}, -\frac{1}{2}, -1, -2, -4$.

Substituting $r = -2$, to ①

$$ar^5 = -8$$

$$a(-2)^5 = -8$$

$$a \times (-32) = -8$$

$$a = \frac{-8}{-32}$$

$$a = \frac{1}{4}$$

First five terms of the geometric progression with $r = -2$ and $a = \frac{1}{4}$ are $\frac{1}{4}, -\frac{1}{2}, 1, -2, 4$.

**Exercise 16.3**

**1.** Find the first term of the geometric progression with common ratio 3 and $4^{th}$ term 108.

**2.** Find the first term of the geometric progression with $6^{th}$ term 1701 and common ratio 3.

**3.** Find the first term of the geometric progression with common ratio $\frac{1}{2}$ and $8^{th}$ term 96.

**4.** The first term of a geometric progression is 5. The $4^{th}$ term is 135. Find the common ratio of the progression.

**5.** The common ratio of a geometric progression is 2 and its first term is 7. Which term is equal to 448?

**6.** The common ratio of a geometric progression is 2 and the first term is $\frac{1}{32}$. Which term is equal to 256?

**7.** Which term is equal to $3\frac{5}{9}$ of the geometric progression with first term 27 and common ratio $\frac{2}{3}$ ?

**8.** Write down the first five terms of the geometric progression with first term 8 and $6^{th}$ term $-256$.

**9.** Show that there are two geometric progressions with first term 64 and ninth term $\frac{1}{4}$, and write the first three terms of each such progression.

**10.** If the 4th term of a geometric progression is 48 and the 7th term is 384, then find the common ratio and the first term of the progression.

**11.** Show that there are two geometric progressions with 3rd term – 45 and 5th term – 1125.

**12.** The 4th term of a geometric progression is 100 and the 9th term is $3\frac{1}{8}$. Write the first five terms of the progression.

**13.** Show that there are two geometric progressions with fifth term as 40 and 9th term as 640 and write the first five terms of each of the progressions.

## 16.4 The sum of the first *n* terms of a geometric progression

The sum of the first $n$ terms of a geometric progression with first term $a$ and common ratio $r$ is denoted by $S_n$. Let us now consider finding a formula for $S_n$.

We can write the first $n$ terms of the geometric progression as,

$$T_1 = a,\ T_2 = ar,\ T_3 = ar^2,\ T_4 = ar^3,\ \ldots,\ T_n = ar^{(n-1)}.$$

$$S_n = T_1 + T_2 + T_3 + T_4 + \ldots + T_n$$

$$\therefore\ S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{(n-1)} \text{ ——— ①}.$$

We use the following technique to find a formula for $S_n$. First we will multiply both sides of equation ① by $r$. Then we get,

$$r S_n = ar + ar^2 + ar^3 + ar^4 + \ldots + ar^n \text{ ——— ②}.$$

Now, when we subtract ① from ②, we get,

$r S_n - S_n = ar^n - a$ (observe that many terms on the right hand side cancel)

$$\therefore\ S_n (r-1) = a(r^n - 1)$$

$$\therefore\ S_n = \frac{a(r^n - 1)}{(r-1)} \quad (r \neq 1)$$

This is an expression for $S_n$ in terms of $a, r, n$. By multiplying both the numerator and the denominator of the right hand side by – 1, we can also express it as

$$S_n = \frac{a(1 - r^n)}{(1 - r)}$$

To find $S_n$, one can use either $S_n = \dfrac{a\,(r^n - 1)}{(r-1)}$ or $S_n = \dfrac{a\,(1-r^n)}{(1-r)}$ appropriately.

**Example 1**

Find the sum of the first five terms of the geometric progression 2, 6, 18, ..., both by finding the first five terms and adding them up, and by using the formula for $S_n$ .

Let us first find the sum by adding all five terms. We are given that

$T_1 = 2,\ T_2 = 6$ and $T_3 = 18$

Moreover,

$T_4 = 18 \times 3 = 54$ and

$T_5 = 54 \times 3 = 162$ .

Therefore, $S_5 = T_1 + T_2 + T_3 + T_4 + T_5$

$$= 2 + 6 + 18 + 54 + 162$$
$$= 242$$

Now let us use $S_n = \dfrac{a\,(r^n - 1)}{(r-1)}$ to find the sum.

Because $a = 2,\ r = \dfrac{6}{2} = 3,\ n = 5.$

and $$S_n = \frac{a\,(r^n - 1)}{r - 1}$$

$$S_5 = \frac{2\,(3^5 - 1)}{3 - 1}$$

$$= \frac{2\,(243 - 1)}{2}$$

$$= \frac{2 \times 242}{2}$$

$$= 242$$

Sum of the first five terms is 242.

When the values of the terms are large or when there are many terms to add, it is easier to to use the formula for $S_n$.

**Example 2**

Find the sum of the first 6 terms of the geometric progression 120, – 60, 30, .... using the formula.

$$a = 120,\ r = \frac{-60}{120} = -\frac{1}{2},\ n = 6$$

Substituting into $S_n = \dfrac{a(1 - r^n)}{1 - r}$ gives us

$$S_6 = \frac{120\left[1 - \left(-\frac{1}{2}\right)^6\right]}{1 - \left(-\frac{1}{2}\right)}$$

$$= \frac{120\left[1 - \left(\frac{1}{64}\right)\right]}{\left(\frac{3}{2}\right)}$$

$$= \left[120 \times \frac{63}{64}\right] \div \frac{3}{2}$$

$$= \left[120 \times \frac{63}{64}\right] \times \frac{2}{3}$$

$$= \frac{315}{4}$$

$$= 78\frac{3}{4}$$

Therefore, the sum of the first six terms is $78\frac{3}{4}$.

There are four unknowns in the formula $S_n = \dfrac{a(1 - r^n)}{1 - r}$. These are $a, r, n$ and $S_n$. When any three of these are given, we can find the value of the remaining unknown by using this formula. Let us consider some examples of this type.

**Example 3**

Find how many terms of the initial terms need to be added from the geometric progression 5, 15, 45, ... in order for the sum to be equal to 1820.

$$a = 5,\ r = \frac{15}{5} = 3,\ S_n = 1820$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$1820 = \frac{5\,(3^n - 1)}{3 - 1}$$

$$1820 = \frac{5\,(3^n - 1)}{2}$$

$$2 \times 1820 = 5\,(3^n - 1)$$

$$\frac{2 \times 1820}{5} = 3^n - 1$$

$$728 = 3^n - 1$$

$$1 + 728 = 3^n$$

$$729 = 3^n$$

$$3^6 = 3^n$$

$$n = 6$$

First six terms need to be added.

**Exercise16.4**

**1.** Find the sum of the first 5 terms of the geometric progression with first term 4 and common ratio 3, both by writing the first 5 terms and adding them up and also using the formula.

**2.** Find the sum of the first 5 terms of the geometric progression 2, 8, 32, ... .

**3.** Find the sum of the first 6 terms of the geometric progression with first term 72 and common ratio $\frac{1}{3}$.

**4.** Find the sum of the first 7 terms of the geometric progression 3, – 6, 12, ... .

**5.** Find the sum of the first 6 terms of the geometric progression 18, 12, 8, ... .

**6.** Show that the sum of the first 6 terms of the geometric progression 18, 6, 2, ... is $26\frac{26}{27}$ .

**7.** How many of the initial terms should be added from the geometric progression 2, 4, 8, ... for the sum to be equal to 2046.

**8.** How many of the initial terms should be added from the geometric progression with first term 4 and common ratio 2 for the sum to be equal to 1020.

**9.** Find the number of initial terms that need to be added from the geometric progression 3, – 12, 48, ... for the sum to be equal to 9831.

## 16.5 Problem solving related to geometric progressions

In this section we will discuss various types of problems related to geometric progressions, that we have not discussed above.

**Example 1**

Of a geometric progression, the sum of the first and second terms is equal to 9 and the sum of the 4th and 5th terms is equal to – 72. Find the first 5 terms of the progression.

$T_1 = a,\ T_2 = ar$

$$a + ar = 9$$
$$a(1 + r) = 9 \quad \text{——} ①$$

$T_4 = ar^3,\ T_5 = ar^4$

$$ar^3 + ar^4 = -72$$
$$ar^3(1 + r) = -72 \quad \text{——} ②$$

② ÷ ①
$$\frac{ar^3(1+r)}{a(1+r)} = \frac{-72}{9}$$
$$r^3 = -8$$
$$r^3 = (-2)^3$$
$$r = -2$$

Substituting $r = -2$, to ①

$$a[1 + (-2)] = 9$$
$$a \times (-1) = 9$$
$$a = -9$$

First five terms of the progression are

$-9, 18, -36, 72, -144.$

**Example 2**

The first three terms of a geometric progression are respectively $(x + 2)$, $(x + 12)$, $(x + 42)$. Find the first term and the common ratio of the progression.

$$r = \frac{x+12}{x+2} = \frac{x+42}{x+12}$$

$$\frac{x+12}{x+2} = \frac{x+42}{x+12}$$

$(x + 12)(x + 12) = (x + 2)(x + 42)$

$$x^2 + 24x + 144 = x^2 + 44x + 84$$
$$144 - 84 = 20x$$
$$60 = 20x$$
$$x = \frac{60}{20}$$
$$x = 3$$

First 3 terms of the sequence are

$(3 + 2), (3 + 12), (3 + 42)$

$5, 15, 45$

First term of the sequence $= 5$

Common ratio of the sequence $= \frac{15}{5}$

$= 3$

**Exercise 16.5**

**1.** The sum of the second and third terms of a geometric progression is 21. Sum of the fifth and sixth terms is 168. Find the first 5 terms of the progression.

**2.** The first three terms of a geometric progression are respectively $4, (x + 3)$ and $(x + 27)$.
(i) Find the value of $x$.
(ii) Show that there are two such geometric progressions and find the first 4 terms of each progression.

**3.** The sum of the first $n$ terms of a progression is given by $4\ (3^n - 1)$.
(i) Show that this sequence is a geometric progression.
(ii) Find the first 4 terms.

**4.** The first three terms of a geometric progression are such that they are the first, third and sixth terms of an arithmetic progression. The fifth term of the arithmetic progression is 15. Find the first 4 terms of the geometric progression.

**5.** The $n^{\text{th}}$ term of a progression is $3(2)^{n+1}$.
(i) Show that this is a geometric progression.
(ii) Find the first term and the common ratio of the progression.

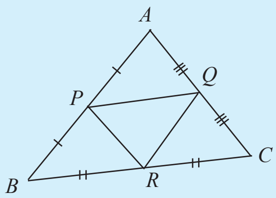
**6.** The first term of a geometric progression is 9. The sum of the first three terms of the progression is 7.
(i) Show that there are two such geometric progressions.
(ii) Write the first four terms of each progression.

## Review Exercise – Term 2

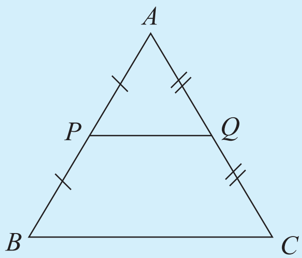
**Part 1**

1. For the collection of numbers 5, 3, 7, 13, 11, 9, 7, 10, 2, 3, 7 write down the following.

   (i) The mode (ii) The median (iii) The mean (iv) The inter quartile range

2. 


   If the perimeter of the triangle $ABC$ is 24 cm, what is the perimeter of the triangle $PQR$?

3. 


   In the triangle $ABC$ the midpoints of the sides $AB$ and $AC$ are $P$ and $Q$ respectively. If the perimeter of triangle $APQ$ is 21 cm, what is the perimeter of triangle $ABC$?

4. A businessman who invests in the stock market bought shares in a certain company when the market price was Rs 50 per share. Later he sold all the shares when the market price was Rs 58 per share. Find the capital gain percentage of the investment the businessman made.

5. An item could be bought for Rs 15 000 by paying cash. Kavindu purchased this item on an installment basis, where the installments were calculated on the reducing balance. She made an initial payment of Rs 3000 and paid off the balance in ten equal monthly installments of Rs 1464. Find the total amount that Kavindu paid for the item.

6. $x = 2$ is one root of the equation $x^2 - ax + 18 = 10$.

   (i) Find the value of $a$.

   (ii) Find the other root of the equation.

7. Find the solutions of the equation $(x - 2)^2 = x - 2$.
8. Solve $3x^2 - 27 = 0$.
9. The sum of the squares of two successive positive integers is 145. Find the two numbers.
10. Determine the following without drawing the graph of the function $y = x^2 + 6x + 5$.
    (i) The axis of symmetry.
    (ii) The minimum value of the function.

11. Write down the $x$ coordinates of the points of intersection of the graph of the function $y = (x - 2)(x + 1)$ and the $x$ - axis.

12. If $\frac{2}{x} + \frac{1}{y} = \frac{5}{6}$ and $\frac{2}{x} - \frac{1}{y} = \frac{1}{6}$, find the values of $x$ and $y$.

13. With reasons, determine what type of progression has $T_n = 2 \times 3^n$ as its $n^{th}$ term.

14. $AB = 6$ cm, $BC = 7$ cm and $AC = 4$ cm in the triangle $ABC$. $x$ is a variable point on the side $BC$. If the midpoint of $AX$ is $P$, describe the locus of $P$.

15. (i) (ii)


Show that ,

the pair of triangles $ABE$ and $ADC$ in figure (i) are equiangular.
the pair of triangles $PQS$ and $QSR$ in figure (ii) are equiangular.

**Part II**

1. When the length of a certain rectangle is reduced by 6 units and the breadth is increased by 2 units, the area of the new rectangle formed is 12 square units less than the area of the original rectangle. By taking the length and the breadth of the initial rectangle as $x$ and $y$ respectively, answer the following.

    (i) Express the length and the breadth of the second rectangle in terms of $x$ and $y$.

(ii) Express the area of the second rectangle in terms of $x$ and $y$.
(iii) Construct an equation in terms *of* $x$ and $y$.
(iv) Show that the length of the initial rectangle is three times its breadth.
(v) If the area of the original rectangle is 192 square units, find its length and breadth.

2. The third term of a geometric progression with a positive common ratio is 3 more than the second term, and the fifth term of the progression is 12 more than the fourth term.

(i) Find the common ratio and the initial term of the progression.
(ii) Write down the first five terms of the progression.
(iii) Show that the $n^{\text{th}}$ term of the progression is $3 \times 2^{n-2}$.

3. A person who invests in the stock market bought 5000 shares in company A which pays annual dividends of Rs 1.25 per share, and a certain number of shares in company B which pays annual dividends of Rs 1.50 per share. When the market price of a share in company A and company B were Rs 30 and Rs 35 respectively, he sold all the shares he owned in the two companies and bought shares in company C at the market price of Rs 50 per share. Company C pays annual dividends of Rs 2.50 per share. His annual dividends income from the investment in company C was Rs 12 750.

(i) Find the number of shares he owned in company *B*.
(ii) Show that the investment in company C resulted in an increase of Rs 2000 in his annual dividends income.

4. A person took a loan of Rs 10 000 at an annual compound interest rate of 8% on the assurance of settling the loan in two years. However, he was unable to pay back the loan in two years as promised. He paid Rs 6000 to the money lender at the end of the second year and came to an agreement with him to pay off the remaining loan amount together with the interest by the end of the following year. This agreement was reached on the condition that he would pay a higher interest for that year.

(i) Calculate the interest for the first year.
(ii) Find the total amount that he needed to pay at the end of the second year to settle the loan.
(iii) How much remained to be paid off at the beginning of the third year?
(iv) If he settled the loan as promised at the end of the third year by paying Rs 6230.40, find the interest rate that was charged for the third year.

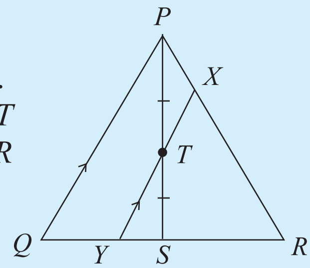
5. The straight line drawn through $B$, parallel to the diagonal $AC$ of the parallelogram $ABCD$, meets the side $DC$ produced at $E$. The straight lines $AE$ and $BC$ intersect at $P$, and the diagonals $AC$ and $BD$ intersect at $Q$.

(i) Draw a sketch and mark the given information.

(ii) Prove that $ABEC$ is a parallelogram.

(iii) Prove that $PQ = \frac{1}{4} DE$ .

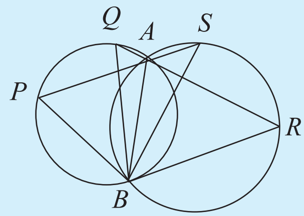
6. The midpoint of the side $QR$ of the triangle $PQR$ is $S$. The midpoint of $PS$ is $T$. The line drawn through $T$ parallel to $PQ$ meets the side $PR$ at $X$ and the side $QR$ at $Y$.




(i) Prove that $YT = \frac{1}{2} PQ$.

(ii) Prove that $XY = \frac{3}{4} PQ$.

7. (a) Based on the information in the figure,

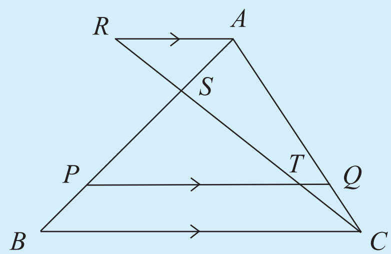



(i) name an angle equal to $A\hat{P}B$ .

(ii) prove that the triangles $BPS$ and $BQR$ are equiangular .

(iii) prove that $BP : BQ = BS : BR$.

(b) Based on the information in the figure,




(i) prove that $\frac{PQ}{BC} = \frac{AQ}{AC}$ .

(ii) prove that $\frac{PQ}{BC} = \frac{RT}{RC}$ .

8. (i) Prepare a table of values to draw the graph of the function $y = x(x - 2)$ in the interval $-3 \leq x \leq 5$.

(ii) Select a suitable scale along the $x$ and $y$ axes and draw the graph of $y = x(x - 2)$.

(iii) By considering the graph,

(a) write down the axis of symmetry of the graph.

(b) the minimum value of the function.

(c) the values of $x$ for which the value of the function is 0.

(d) the roots of the equation $x(x-2)=0$.

(e) the range of values of $x$ for which the function is negative.

(iv) Draw the graph $y=x^2$ and find the value of $\sqrt{2}$ to the first decimal place, using the graph.

## ලඝුගණක

klf;iffs;

LOGARITHMS

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Mean Differences (මධ්‍යන්‍ය අන්තරය / ,il tpj;jahrq;fs;) 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 | 0212 | 0253 | 0294 | 0334 | 0374 | 4 | 8 | 12 | 17 | 21 | 25 | 29 | 33 | 37 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 | 4 | 8 | 11 | 15 | 19 | 23 | 26 | 30 | 34 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 | 3 | 7 | 10 | 14 | 17 | 21 | 24 | 28 | 31 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 | 3 | 6 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 | 3 | 6 | 8 | 11 | 14 | 17 | 20 | 22 | 25 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 | 3 | 5 | 8 | 11 | 13 | 16 | 18 | 21 | 24 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 | 2 | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 22 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 | 2 | 4 | 7 | 9 | 11 | 13 | 16 | 18 | 20 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 | 2 | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 22 | 3424 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 15 | 17 |
| 23 | 3617 | 3636 | 3655 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3784 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 | 2 | 4 | 5 | 7 | 9 | 11 | 12 | 14 | 16 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 | 2 | 3 | 5 | 7 | 9 | 10 | 12 | 14 | 15 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 | 2 | 3 | 5 | 7 | 8 | 10 | 11 | 13 | 15 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 13 | 14 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 29 | 4624 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 12 | 13 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 11 | 13 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 12 |
| 32 | 5051 | 5065 | 5079 | 5092 | 5105 | 5119 | 5132 | 5145 | 5159 | 5172 | 1 | 3 | 4 | 5 | 7 | 8 | 9 | 11 | 12 |
| 33 | 5185 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 | 1 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 12 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 | 1 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 11 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5514 | 5527 | 5539 | 5551 | 1 | 2 | 4 | 5 | 6 | 7 | 9 | 10 | 11 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 5635 | 5647 | 5658 | 5670 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 | 1 | 2 | 3 | 4 | 5 | 7 | 8 | 9 | 10 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 44 | 6435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 45 | 6532 | 6542 | 6551 | 6561 | 6571 | 6580 | 6590 | 6599 | 6609 | 6618 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 8 |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 49 | 6902 | 6911 | 6920 | 6928 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 7 |
| 53 | 7243 | 7251 | 7259 | 7267 | 7275 | 7284 | 7292 | 7300 | 7308 | 7316 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| 54 | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

# ලඝුගණක

## மடக்கைகள்

## LOGARITHMS

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | මධ්‍යන්‍ය අන්තරය / இடை வித்தியாசங்கள் / Mean Differences 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 55 | 7404 | 7412 | 7419 | 7427 | 7435 | 7443 | 7451 | 7459 | 7466 | 7474 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 56 | 7482 | 7490 | 7497 | 7505 | 7513 | 7520 | 7528 | 7536 | 7543 | 7551 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 57 | 7559 | 7566 | 7574 | 7582 | 7589 | 7597 | 7604 | 7612 | 7619 | 7627 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 58 | 7634 | 7642 | 7649 | 7657 | 7664 | 7672 | 7679 | 7686 | 7694 | 7701 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 59 | 7709 | 7716 | 7723 | 7731 | 7738 | 7745 | 7752 | 7760 | 7767 | 7774 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 60 | 7782 | 7789 | 7796 | 7803 | 7810 | 7818 | 7825 | 7832 | 7839 | 7846 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 61 | 7853 | 7860 | 7868 | 7875 | 7882 | 7889 | 7896 | 7903 | 7910 | 7917 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 62 | 7924 | 7931 | 7938 | 7945 | 7952 | 7959 | 7966 | 7973 | 7980 | 7987 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| 63 | 7993 | 8000 | 8007 | 8014 | 8021 | 8028 | 8035 | 8041 | 8048 | 8055 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 64 | 8062 | 8069 | 8075 | 8082 | 8089 | 8096 | 8102 | 8109 | 8116 | 8122 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 65 | 8129 | 8136 | 8142 | 8149 | 8156 | 8162 | 8169 | 8176 | 8182 | 8189 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 66 | 8195 | 8202 | 8209 | 8215 | 8222 | 8228 | 8235 | 8241 | 8248 | 8254 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 67 | 8261 | 8267 | 8274 | 8280 | 8287 | 8293 | 8299 | 8306 | 8312 | 8319 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 68 | 8325 | 8331 | 8338 | 8344 | 8351 | 8357 | 8363 | 8370 | 8376 | 8382 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| 69 | 8388 | 8395 | 8401 | 8407 | 8414 | 8420 | 8426 | 8432 | 8439 | 8445 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 70 | 8451 | 8457 | 8463 | 8470 | 8476 | 8482 | 8488 | 8494 | 8500 | 8506 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 71 | 8513 | 8519 | 8525 | 8531 | 8537 | 8543 | 8549 | 8555 | 8561 | 8567 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 72 | 8573 | 8579 | 8585 | 8591 | 8597 | 8603 | 8609 | 8615 | 8621 | 8627 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 73 | 8633 | 8639 | 8645 | 8651 | 8657 | 8663 | 8669 | 8675 | 8681 | 8686 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 74 | 8692 | 8698 | 8704 | 8710 | 8716 | 8722 | 8727 | 8733 | 8739 | 8745 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 75 | 8751 | 8756 | 8762 | 8768 | 8774 | 8779 | 8785 | 8791 | 8797 | 8802 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 76 | 8808 | 8814 | 8820 | 8825 | 8831 | 8837 | 8842 | 8848 | 8854 | 8859 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 77 | 8865 | 8871 | 8876 | 8882 | 8887 | 8893 | 8899 | 8904 | 8910 | 8915 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 78 | 8921 | 8927 | 8932 | 8938 | 8943 | 8949 | 8954 | 8960 | 8965 | 8971 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 79 | 8976 | 8982 | 8987 | 8993 | 8998 | 9004 | 9009 | 9015 | 9020 | 9025 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 80 | 9031 | 9036 | 9042 | 9047 | 9053 | 9058 | 9063 | 9069 | 9074 | 9079 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 81 | 9085 | 9090 | 9096 | 9101 | 9106 | 9112 | 9117 | 9122 | 9128 | 9133 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 82 | 9138 | 9143 | 9149 | 9154 | 9159 | 9165 | 9170 | 9175 | 9180 | 9186 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 83 | 9191 | 9196 | 9201 | 9206 | 9212 | 9217 | 9222 | 9227 | 9232 | 9238 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 84 | 9243 | 9248 | 9253 | 9258 | 9263 | 9269 | 9274 | 9279 | 9284 | 9289 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 85 | 9294 | 9299 | 9304 | 9309 | 9315 | 9320 | 9325 | 9330 | 9335 | 9340 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 86 | 9345 | 9350 | 9355 | 9360 | 9365 | 9370 | 9375 | 9380 | 9385 | 9390 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 87 | 9395 | 9400 | 9405 | 9410 | 9415 | 9420 | 9425 | 9430 | 9435 | 9440 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 88 | 9445 | 9450 | 9455 | 9460 | 9465 | 9469 | 9474 | 9479 | 9484 | 9489 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 89 | 9494 | 9499 | 9504 | 9509 | 9513 | 9518 | 9523 | 9528 | 9533 | 9538 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 90 | 9542 | 9547 | 9552 | 9557 | 9562 | 9566 | 9571 | 9576 | 9581 | 9586 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 91 | 9590 | 9595 | 9600 | 9605 | 9609 | 9614 | 9619 | 9624 | 9628 | 9633 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 92 | 9638 | 9643 | 9647 | 9652 | 9657 | 9661 | 9666 | 9671 | 9675 | 9680 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 93 | 9685 | 9689 | 9694 | 9699 | 9703 | 9708 | 9713 | 9717 | 9722 | 9727 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 94 | 9731 | 9736 | 9741 | 9745 | 9750 | 9754 | 9759 | 9763 | 9768 | 9773 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 95 | 9777 | 9782 | 9786 | 9791 | 9795 | 9800 | 9805 | 9809 | 9814 | 9818 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 96 | 9823 | 9827 | 9832 | 9836 | 9841 | 9845 | 9850 | 9854 | 9859 | 9863 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 97 | 9868 | 9872 | 9877 | 9881 | 9886 | 9890 | 9894 | 9899 | 9903 | 9908 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 98 | 9912 | 9917 | 9921 | 9926 | 9930 | 9934 | 9939 | 9943 | 9948 | 9952 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 99 | 9956 | 9961 | 9965 | 9969 | 9974 | 9978 | 9983 | 9987 | 9991 | 9996 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

**A**

| | |
|---|---|
| Axis of symmetry | සමමිති අක්ෂය |

**C**

| | |
|---|---|
| Class Boundaries | පන්ති මායිම් |
| Class intervals | පන්ති ප්‍රාන්තර |
| Class Limits | පන්ති සීමා |
| Class width | පන්තියක තරම |
| Coefficient | සංගුණකය |
| Common Ratio | පොදු අනුපාතය |
| Completing the Square | වර්ග පූර්ණය |
| Compound Interest | වැල් පොලිය |
| Continuous data | සන්තතික දත්ත |
| Converse | විලෝමය |
| Cumulative Frequency | සමුච්චිත සංඛ්‍යාතය |
| Cumulative Frequency | සමුච්චිත සංඛ්‍යාතය වක්‍රය |

**D**

| | |
|---|---|
| Data | දත්ත |
| Discrete data | විවික්ත දත්ත |
| Domain | වසම |

**F**

| | |
|---|---|
| First Term | පළමුවන පදය |
| Frequency | සංඛ්‍යාතය |
| Frequency polygon | සංඛ්‍යාත බහුඅස්‍රය |
| Function | ශ්‍රිතය |

**G**

| | |
|---|---|
| Geometric progression | ගුණෝත්තර ශ්‍රේඪී |

**H**

| | |
|---|---|
| Histogram | ජාල රේඛය |

**I**

| | |
|---|---|
| Instalment | වාරිකය |

**M**

| | |
|---|---|
| Maximum value | උපරිම අගය |
| Mid point | මධ්‍ය ලක්ෂ්‍ය |
| Minimum value | අවම අගය |

**N**

| | |
|---|---|
| Number of month units | මාස ඒකක ගණන |
| Number Sequence | සංඛ්‍යා අනුක්‍රම |

**P**

| | |
|---|---|
| Preceding Term | පෙර පදය |
| Proof | සාධනය |
| Proportional | සමානුපාතික |

**Q**

| | |
|---|---|
| Quadratic Equation | වර්ගජ සමීකරණ |

**R**

| | |
|---|---|
| Range | පරාසය / ප්‍රාන්තරය |
| Reducing Balance | හීනවන ශේෂය |
| Rider | අනුමේය |

**S**

| | |
|---|---|
| Simultaneous equations | සමගාමී සමීකරණ |
| Solutions | විසඳුම |
| Successive Term | පසු පදය |

**T**

| | |
|---|---|
| Turning point | හැරුම් ලක්ෂ්‍යය |

**U**

| | |
|---|---|
| Unknown | අඥාතය |

**V**

| | |
|---|---|
| Verification | සත්‍යාපනය |

## Sequence of the Lessons

| Chapter of Textbook | No.of Periods |
|---|---|
| **1 Term** | |
| 1. Real Numbers | 10 |
| 2. Indices and Logarithms I | 08 |
| 3. Indices and Logarithms II | 06 |
| 4. Surface Area of Solids | 05 |
| 5. Volume of the Solids | 05 |
| 6. Binomial Expressions | 04 |
| 7. Algebraic Fractions | 04 |
| 8. Areas of Plane Figures between Parallel Lines | 12 |
| | |
| **2 Term** | |
| 09. Percentages | 06 |
| 10. Share Market | 05 |
| 11. Mid Point Theorem | 05 |
| 12. Graphs | 12 |
| 13. Formulae | 10 |
| 14. Equiangular Triangles | 12 |
| 15. Data representaion and Interpretation | 12 |
| 16. Geometric Progressions | 06 |
| | |
| **3 Term** | |
| 17. Pythagoras's Theorem | 04 |
| 18. Trignometry | 12 |
| 19. Matrices | 08 |
| 20. Inequalities | 06 |
| 21. Cyclic Quadrilaterals | 10 |
| 22. Tangent | 10 |
| 23. Constructions | 05 |
| 24. Sets | 06 |
| 25. Probability | 07 |